Per Unit Normalization

Purpose:
1. makes problems easier to work with
2. comparison of equipments – certain parameters (eg impedances of sync gen) vary from unit to unit, making comparisons difficult
3. removes xform turns ratio from analysis
   – replace "simplified per unit diagram" by "impedance diagram"

Definition:

$$\text{quantity in per unit} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

Notation:

p.u.	$X = 0.1$ pu
percent	$X = 10\%$

Base Quantities: all real numbers (magnitudes)

Voltage	V_B
Complex Power	S_B
Current	I_B
Impedance	Z_B
Admittance	$Y_B = \frac{1}{Z_B}$

If system is normal, transformer induced phase shifts can be eliminated in pu analysis
" " " not normal, " " " must be included in pu analysis

Rules

Single Phase

1. Base values must satisfy ohm's law & power equation

a. $V_B = Z_B I_B$

$$\frac{\bar{V}}{V_B} = \frac{\bar{Z}}{Z_B}\frac{\bar{I}}{I_B} \Rightarrow \quad \bar{V}_{p.u.} = \bar{Z}_{pu}\bar{I}_{pu}$$

b $S_B = V_B I_B$

$$\frac{\bar{S}}{S_B} = \frac{\bar{V}}{V_B}\frac{\bar{I}^*}{I_B} \Rightarrow \quad \bar{S}_{pu} = \bar{V}_{pu}\bar{I}_{pu}^*$$

$$\frac{\bar{S}}{S_B} = \frac{P}{S_B} + j\frac{Q}{S_B} \qquad \bar{S}_{pu} = P_{pu} + j\,Q_{pu}$$

2. There are 2 independent base values (1ϕ)

$$\left.\begin{array}{l} V_B = Z_B I_B \\ S_B = V_B I_B \end{array}\right\} \text{2 equations in 4 unknowns}$$

Normally select V_B & S_B, and then solve for I_B & Z_B

S_B is held fixed for entire system

V_B changes (due to transformers)

$$Z_B = \frac{V_B^2}{S_B}$$

$$I_B = \frac{S_B}{V_B}$$

Rules (cont'd) 11/4/02

3. Independent Base Values (3ϕ)

$$\underbrace{S_B^{3\phi}}_{\text{Base kVA}_{3\phi}} \triangleq 3 \underbrace{S_B}_{\text{Base kVA}_{1\phi}} \tag{5.27}$$

$$\underbrace{V_B^{ll}}_{\text{Base kV}_{LL}} \triangleq \sqrt{3}\, \underbrace{V_B}_{\text{Base kV}_{LN}} \tag{5.28}$$

where

V_B = per phase base voltage (line to neutral)
S_B = " " " power
V_B^{ll} = 3 ϕ base voltage

4. PU Quantities

a) $$\underbrace{\bar{S}_{PU}^{3\phi}}_{3\phi \text{ power in pu}} = \frac{\bar{S}^{3\phi}}{S_B^{3\phi}} = \frac{3\bar{S}}{3S_B} = \underbrace{\bar{S}_{PU}}_{1\phi \text{ power in pu}} \tag{5.29}$$

Note: $\bar{S}_{PU}^{3\phi} = \bar{S}_{PU}$

b) Voltage

$$\bar{V}_{pu}^{ll} = \frac{\overbrace{\bar{V}^{ll}}^{\text{line to line voltage}}}{V_B^{ll}} = \frac{\sqrt{3}\, \overbrace{\bar{V}^{ln}}^{\text{line to neutral voltage}}}{\sqrt{3}\, V_B} = \bar{V}_{pu} \tag{5.30}$$

Note: $\bar{V}_{pu}^{ll} = \bar{V}_{pu}$

c) Impedance

$$Z_B = \frac{(V_B^{ll})^2}{S_B^{3\phi}} = \frac{(\sqrt{3}\,V_B)^2}{3S_B} = \frac{V_B^2}{S_B} = Z_B \tag{5.31}$$

∴ Z_B is same whether calc using 1ϕ or 3ϕ base values

d) Current

$$S_B^{3\phi} = 3 V_B I_B = \sqrt{3} V_B^{ll} I_B$$

$$I_B = \frac{S_B^{3\phi}}{\sqrt{3}V_B^{ll}} = \frac{3S_B^{1\phi}}{\sqrt{3}(\sqrt{3}V_B)} = \frac{S_B^{1\phi}}{V_B} = I_B$$

∴ I_B is same whether calc using 1ϕ or 3ϕ base values

Rules (cont'd) 8/24/84

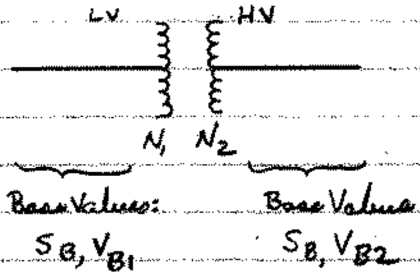
5. Transformers

general approach: choose different base voltage values on each side of xformer

rule: Ratio of Voltage Bases = Connection Induced Voltage Magnitude Ratio in the per phase equivalent ckt

a) Y-Y

see pg 132-133 in Bergen

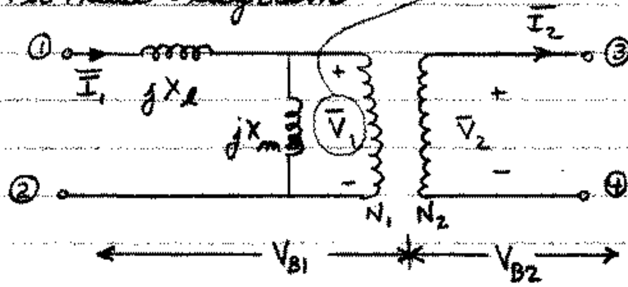


let X_ℓ = leakage reactance
X_m = magnetizing reactance

applying rule: $\dfrac{V_{B2}}{V_{B1}} = \dfrac{N_2}{N_1} = n$

then
$$Z_{B1} = \frac{V_{B1}^2}{S_B} \quad \text{and} \quad Z_{B2} = \frac{V_{B2}^2}{S_B}$$

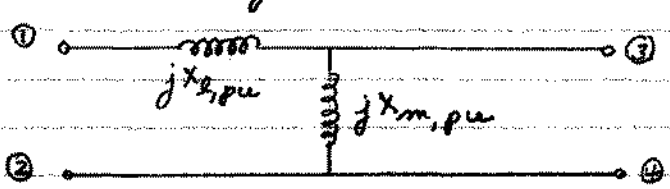
Per Phase Diagram



since
$$\bar{V}_{1,pu} = \frac{\bar{V}_1}{V_{B1}}$$

$$\bar{V}_{2,pu} = \frac{\bar{V}_2}{V_{B2}} = \frac{n\bar{V}_1}{nV_{B1}} = \bar{V}_{1,pu}$$

∴ ideal transformer can be eliminated

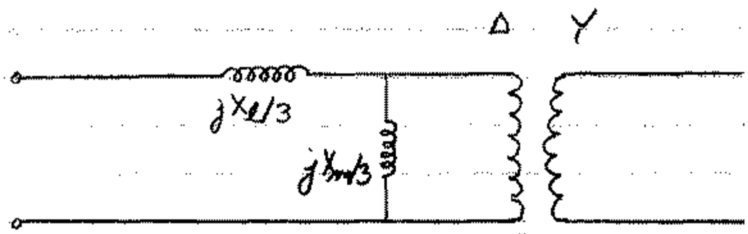


$$jX_{\ell,pu} = \frac{jX_\ell}{Z_{B1}} \qquad jX_{m,pu} = \frac{jX_m}{Z_{B1}}$$

b) $\Delta - Y$

LV HV

Simplified Per Phase Diagram (all phase shifts left out)



Base Values
S_B, V_{B1}

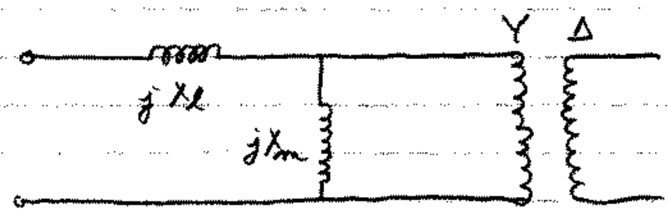
Base Values
S_B, V_{B2}

applying rule

ratio of voltage bases $\Longrightarrow$ $$\frac{V_{B2}}{V_{B1}} = \sqrt{3}\, n \Longleftarrow$$ Connection Induced Voltage Magnitude Ratio in Per Phase Equiv. ckt

c) $Y - \Delta$

LV HV



S_B, V_{B1} S_B, V_{B2}

$$\frac{V_{B2}}{V_{B1}} = |K_3| = \frac{n}{\sqrt{3}}$$

d) $\Delta - \Delta$ $$\frac{V_{B2}}{V_{B1}} = \frac{N_2}{N_1}$$

6. Changing Base

Let

$$Z_{pu}^{old} = \text{pu impedance based on nameplate rating} = \frac{Z^{actual}}{Z_B^{old}}$$

where

$$Z_B^{old} = \frac{(V^{nameplate})^2}{S^{nameplate}} \qquad (V_B^{ll}, S_B^{3\phi}) \text{ or } (V_B, S_B)$$

$$Z^{actual} = \text{impedance before pu normalization}$$

Since all equipment in the system will not have the same nameplate ratings for V & S, we will arbitrarily select the nameplate ratings from one piece of equipment as the reference, and then adjust the pu impedances of the remaining equipment as follows:

$$Z_{pu}^{new} = \frac{Z^{actual}}{Z_B^{new}}$$

where

$$Z_B^{new} = \text{new base impedance}$$

then

$$\begin{aligned} Z_{pu}^{new} &= Z_{pu}^{old}\left[\frac{Z_B^{old}}{Z_B^{new}}\right] \\ &= Z_{pu}^{old}\left[\frac{V_B^{old}}{V_B^{new}}\right]^2 \frac{S_B^{new}}{S_B^{old}} \end{aligned} \qquad (5.33)$$

nameplate value

Per Unit Analysis of Normal Systems

(a) Pick a common three-phase MVA base for the entire system

(b) In one part of the system, arbitrarily pick a line-to-line kV base

(c) Relate the line-to-line kV bases across transformers according to the ratio of nameplate line-to-line kV ratings of 3ϕ transformers

(d) Find the impedance bases in different sections according to the formula

$$Z_B = \frac{(V_B^{ll})^2}{S_B^{3\phi}}$$

(e) Adjust all equipment nameplate per-unit impedances according to the base values.

(f) Draw the impedance diagram for the entire system, and solve for desired per-unit quantities

(g) Convert back to actual quantities if desired

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8/24/04

Example 5.11

Given a 1ϕ transformer

Nameplate Data : 1000 KVA, 13.2 - 66 kV, $X_\ell = 0.1$ pu, $X_m = 100$ pu

a) On 13.2 kV side

$$Z_{1B} = \frac{V_{1B}^2}{S_B} = \frac{(13.2\times10^3)^2}{1000\times10^3} = 174\,\Omega \Rightarrow X_\ell = X_\ell^{pu}\cdot 174 = 0.1\times 174 = 17.4\,\Omega$$

$$X_m = X_m^{pu}\cdot 174 = 100\times 174 = 17{,}400\,\Omega$$

b) On 66 kV side

$$Z_{2B} = \frac{V_{2B}^2}{S_B} = \frac{(66\times10^3)^2}{1000\times10^3} = 4356\,\Omega \Rightarrow X_\ell = 0.1\times 4356 = 435.6\,\Omega$$

$$X_m = 100\times 4356 = 435600\,\Omega$$

c) 3ϕ Connections

1. $\underbrace{Y}$ - Δ or $\underbrace{Y}$ - Y

$$S_B^{3\phi} = 3\times 1000\text{ kva} = 3000\text{ kVa}$$
$$V_B^{\ell\ell} = \sqrt{3}\cdot 13.2\text{ kV}$$

$$Z_B = \frac{(V_B^{\ell\ell})^2}{S_B^{3\phi}} = \frac{(\sqrt{3}\cdot 13.2\text{ kV})^2}{3000\times10^3} = 174\,\Omega$$

$$\therefore X_{\ell pu} = \frac{17.4}{174} = 0.1\text{ pu}$$

$$X_{m pu} = \frac{17{,}400}{174} = 100\text{ pu}$$

2. $\underbrace{\Delta}$ - Y or $\underbrace{\Delta}$ - Δ

$$S_B^{3\phi} = 3000\text{ kVA}$$
$$V_B^{\ell\ell} = 13.2\text{ kV}$$

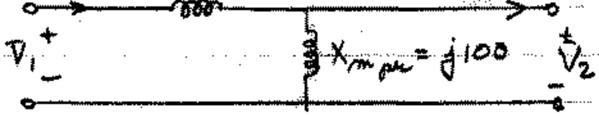
$$Z_{1B} = \frac{(13.2\text{ kV})^2}{3000\times10^3} = 58.1\,\Omega$$

$$\therefore X_{\ell pu} = \frac{(17.4/3)}{58.1} = 0.1\text{ pu}$$

$$X_{m pu} = \frac{17{,}400/3}{58.1} = 100\text{ pu}$$

Important:
As long as we use the nameplate V & S as the basis, then the pu impedances in the "perphase" diagram are the same as the 1ϕ nameplate values.

d)



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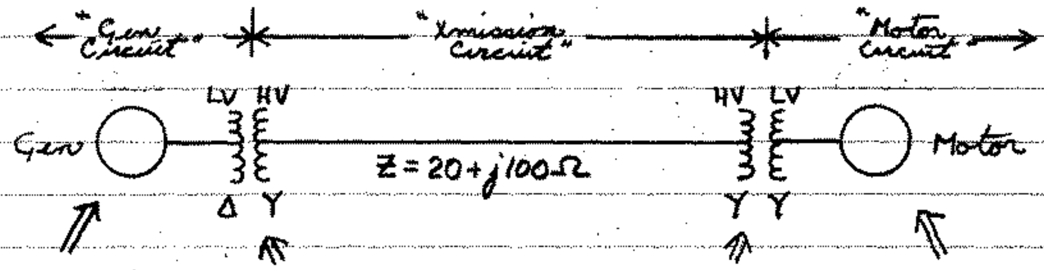
Ex: Draw an impedance diagram for the following system



30MVA(3φ)	20MVA(3φ)	15MVA(3φ)	20MVA(3φ)
13.8kV	13.2–132kV (l-l on Y side)	138–13.8kV	pf = 1.0
$X_s = 0.1\text{pu}$	$X_l = 0.1\text{pu}$ (l-l on Δ side)	$X_l = 0.12\text{pu}$	13.8 kV

Base Values

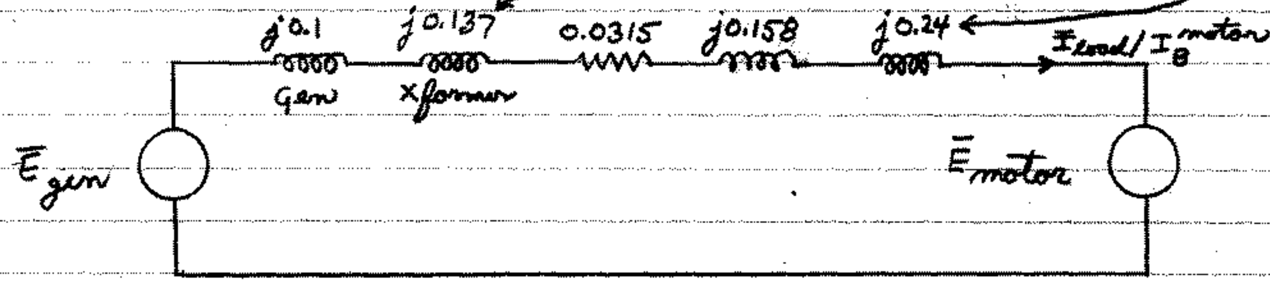
Note: all voltages are line to line

Solution

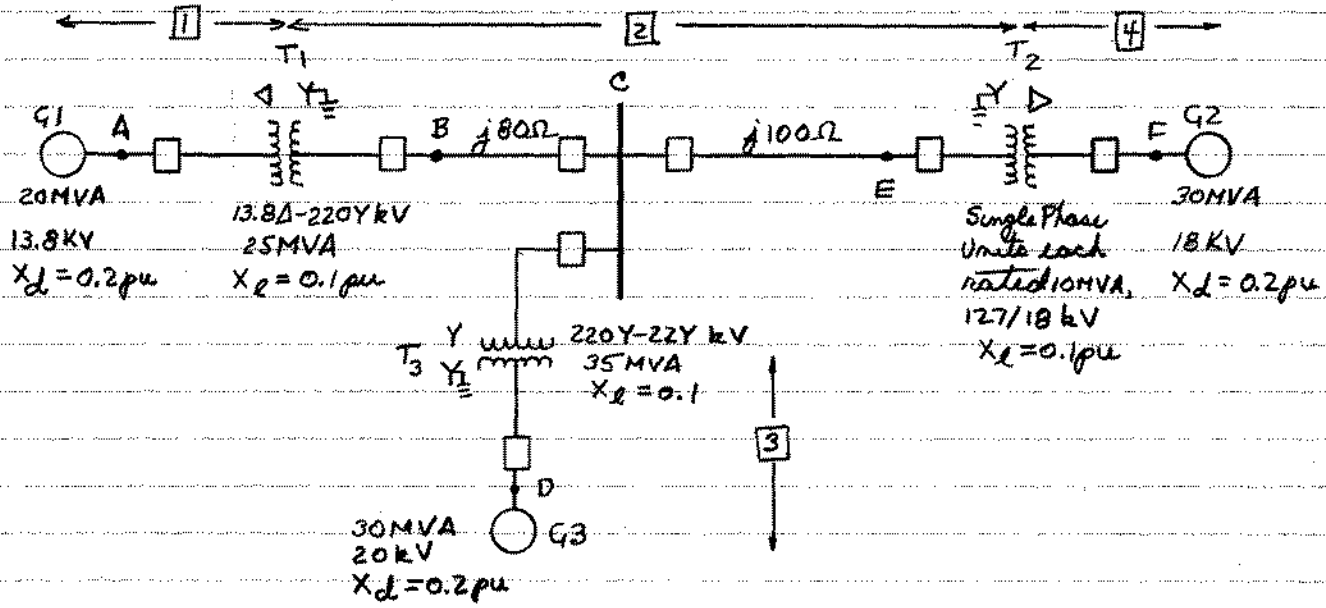
	Gen Circuit	Xmission Circuit	Motor Circuit
$S_B^{3\phi}$ (MVA)	30	30	30
V_B^{ll} (kV)	13.8	$13.8 \times \frac{132}{13.2} = 138$	$138 \times \frac{13.8}{138} = 13.8$
Z_B (Ω)	Not Needed	$\frac{(V_B^{ll})^2}{S_B^{3\phi}} = 634.8\,\Omega$	Not Needed
I_B (A)	$\frac{S_B^{3\phi}}{\sqrt{3}V_B^{ll}} = 1255.1$	$\frac{S_B^{3\phi}}{\sqrt{3}V_B^{ll}} = 125.5$	$\frac{S_B^{3\phi}}{\sqrt{3}V_B^{ll}} = 1255.1$

$$Z_{pu}^{new} = Z_{pu}^{old}\left(\frac{V_B^{old}}{V_B^{new}}\right)^2 \frac{S_B^{new}}{S_B^{old}} = 0.1\left(\frac{13.2}{13.8}\right)^2\left(\frac{30}{20}\right) =$$

$$Z_{pu}^{new} = 0.12\left(\frac{13.8}{13.8}\right)^2\left(\frac{30}{15}\right) =$$

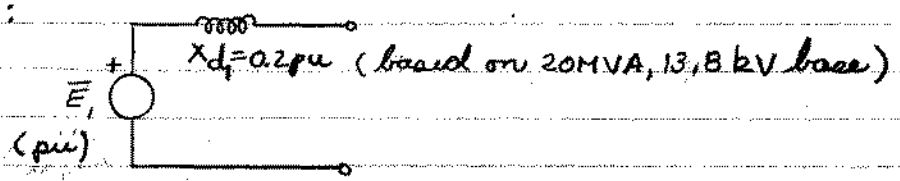


Example: Draw the impedance diagram for the following system. Choose a base of 50MVA and 13.8kV in the circuit of generator 1.



Side $\boxed{1}$: $S_B^{3\phi} = 50\text{MVA}$ $\quad V_B^{ll} = 13.8\text{kV}$

Generator G1:



$$X_{d_1}^{new} = X_{d_1}\left(\frac{V_B^{ll(old)}}{V_B^{ll(new)}}\right)^2\left(\frac{S_B^{new}}{S_B^{old}}\right) = 0.2\left(\frac{13.8}{13.8}\right)^2\left(\frac{50}{20}\right) = 0.5\text{pu}$$

Transformer T1:

X_{l_1} on Δ side = 0.1 pu $\qquad$ (based on 25MVA, 13.8kV base)

$$X_{l_1}^{new} = 0.1\left(\frac{13.8}{13.8}\right)^2\left(\frac{50}{25}\right) = 0.2\text{pu}$$

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Side [2] $S_B^{3\phi} = 50\,MVA$ $V_B^{ll} = 220\,kV$

$$Z_B = \frac{(V_B^{ll})^2}{S_B^{3\phi}} = \frac{220^2}{50} = 968\,\Omega$$

Xmission Line BC: $X_{BC} = \frac{80}{968} = 0.083\,pu$

Xmission Line CE: $X_{EC} = \frac{100}{968} = 0.103\,pu$

Transformer T1: $X_{l_1}^{new} = 0.1\left(\frac{220}{220}\right)^2\left(\frac{50}{25}\right) = 0.2\,pu$ (same as for side [1])

Transformer T2: $X_{l_2}^{new} = 0.1\left(\frac{\sqrt{3}\cdot 127}{220}\right)^2\left(\frac{50}{3\cdot 10}\right) = 0.166\,pu$

Transformer T3: $X_{l_3}^{new} = 0.1\left(\frac{220}{220}\right)^2\left(\frac{50}{35}\right) = 0.143\,pu$

Side [3] $S_B^{3\phi} = 50\,MVA$ $V_B^{ll} = 22\,kV$

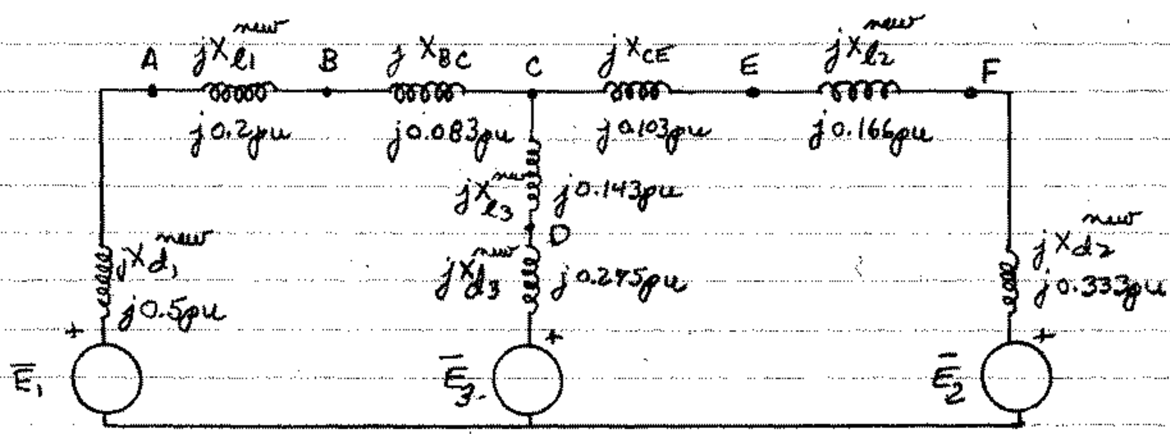
generator G3: $X_{d_3}^{new} = 0.2\left(\frac{20}{22}\right)^2\left(\frac{50}{30}\right) = 0.275\,pu$

Transformer T3: $X_{l_3}^{new} = 0.1\left(\frac{22}{22}\right)^2\left(\frac{50}{35}\right) = 0.143\,pu$ (same as for side [2])

Side [4] $S_B^{3\phi} = 50\,MVA$ $V_B^{ll} = 220 \times \frac{18}{127\sqrt{3}} = 18\,kV$

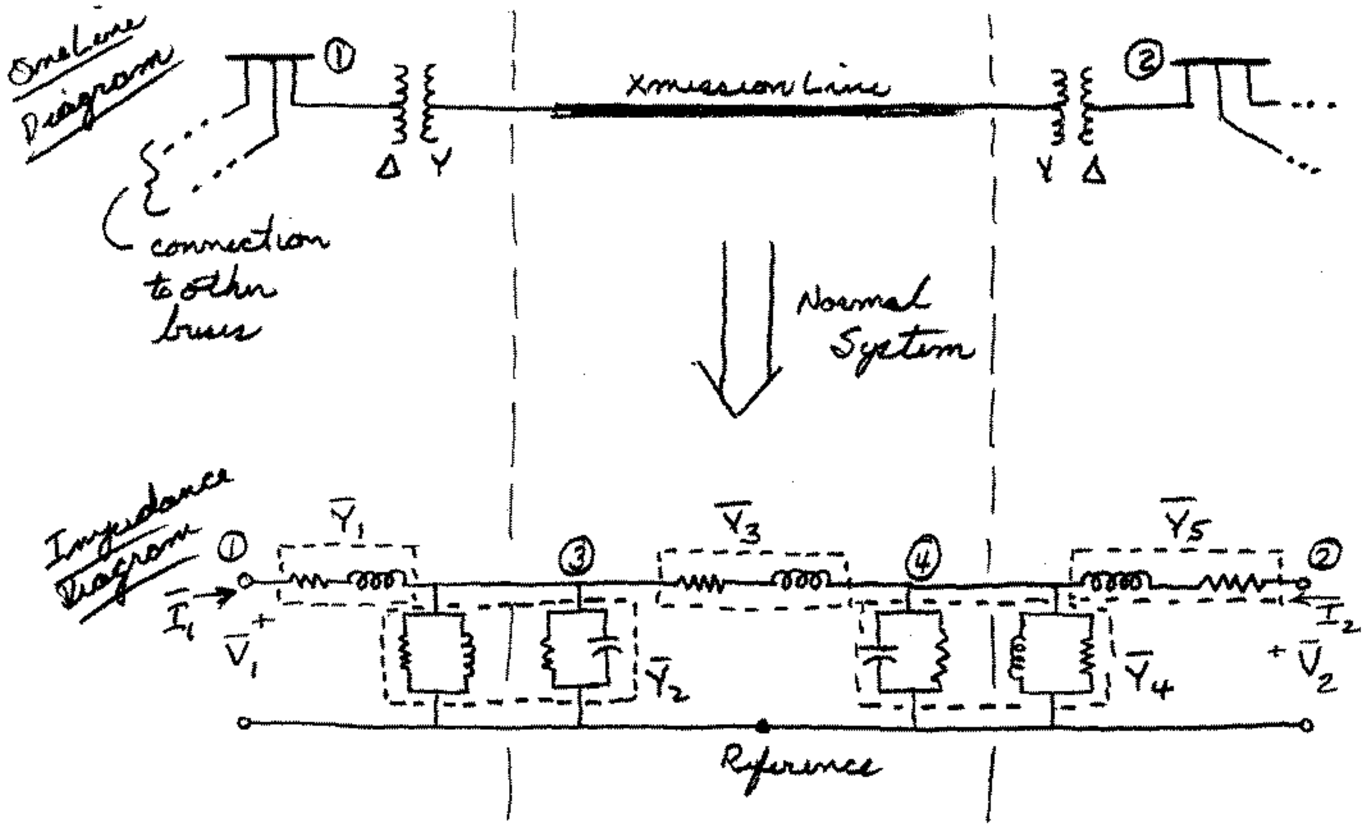
Generator G2: $X_{d2}^{new} = 0.2\left(\frac{18}{18}\right)^2\left(\frac{50}{30}\right) = 0.333\,pu$

Transformer T2: $X_{l2}^{new} = 0.1\left(\frac{18}{18}\right)^2\left(\frac{50}{3.10}\right) = 0.166\,pu$ (same as for side [2])



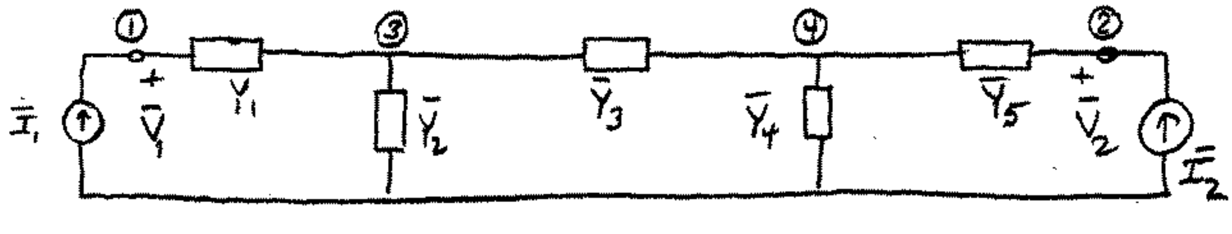
Transmission Lines and Transformers

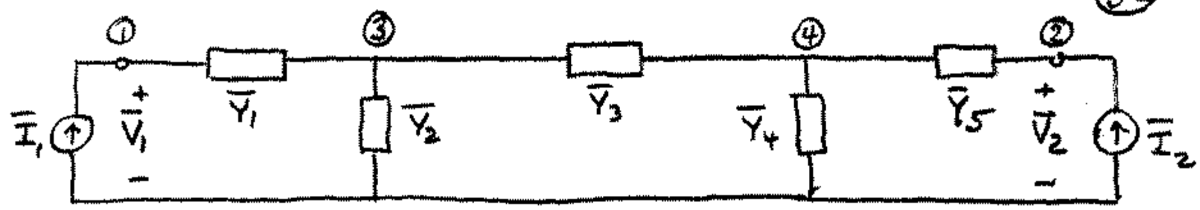
Let us consider a typical transmission line / transformer connection between two buses in a power system



Number of nodes in the Impedance Diagram above? 4 + 1 ← Reference

In power system calculations, the currents $\bar{I}_1$ & $\bar{I}_2$ are often known, and the voltages $\bar{V}_1$ & $\bar{V}_2$ need to be determined. Let us add current sources at the ends of the Impedance Diagram





The node equations :

node	equation
①	$\bar{I}_1 = \bar{Y}_1(\bar{V}_1 - \bar{V}_3)$
②	$\bar{I}_2 = \bar{Y}_5(\bar{V}_2 - \bar{V}_4)$
③	$0 = \bar{Y}_3(\bar{V}_3 - \bar{V}_4) + \bar{Y}_2(\bar{V}_3 - 0) + \bar{Y}_1(\bar{V}_3 - \bar{V}_1)$
④	$0 = \bar{Y}_3(\bar{V}_4 - \bar{V}_3) + \bar{Y}_4(\bar{V}_4 - 0) + \bar{Y}_5(\bar{V}_4 - \bar{V}_2)$

Linear Eq$^{\underline{n}}$

current sources
(connected to reference)

Put in matrix form:

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \hline 0 \\ 0 \end{bmatrix} = \left[\begin{array}{cc|cc} \bar{Y}_1 & 0 & -\bar{Y}_1 & 0 \\ 0 & \bar{Y}_5 & 0 & -\bar{Y}_5 \\ \hline -\bar{Y}_1 & 0 & \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 & -\bar{Y}_3 \\ 0 & -\bar{Y}_5 & -\bar{Y}_3 & \bar{Y}_3 + \bar{Y}_4 + \bar{Y}_5 \end{array}\right] \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \hline \bar{V}_3 \\ \bar{V}_4 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \underline{I} \\ \hline \underline{0} \end{bmatrix}}_{\text{known}} = \underbrace{\left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right]}_{=\bar{Y}} \underbrace{\begin{bmatrix} \underline{V} \\ \hline \underline{V}' \end{bmatrix}}_{\text{unknown}} \qquad (*)$$

A number of methods to solve Eq (*) for $\bar{V}_1, \bar{V}_2, \bar{V}_3, \bar{V}_4$.
One method is to invert $[Y]$.

$$\begin{bmatrix} \underline{V} \\ \underline{V}' \end{bmatrix} = [Z] \begin{bmatrix} \underline{I} \\ \underline{0} \end{bmatrix} \qquad \text{where} \quad [Z] = [Y]^{-1}$$

Alternate Method

- In many power system studies, one is usually interested in determining $\bar{V}_1$ and $\bar{V}_2$. The other voltages ($\bar{V}_3$ and $\bar{V}_4$) can be calculate once $\bar{V}_1$ & $\bar{V}_2$ are known.

- Extract equations for nodes 3 and 4 from Eq (*)

$$\underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\underline{0}} = \underbrace{\begin{bmatrix} -\bar{Y}_1 & 0 \\ 0 & -\bar{Y}_5 \end{bmatrix}}_{[A_{21}]} \underbrace{\begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix}}_{\underline{V}} + \underbrace{\begin{bmatrix} \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 & -\bar{Y}_3 \\ -\bar{Y}_3 & \bar{Y}_3 + \bar{Y}_4 + \bar{Y}_5 \end{bmatrix}}_{[A_{22}]} \underbrace{\begin{bmatrix} \bar{V}_3 \\ \bar{V}_4 \end{bmatrix}}_{\underline{V}'}$$

- Solve for $\underline{V}'$

$$\underline{V}' = -[A_{22}]^{-1}[A_{21}]\underline{V} \qquad (**)$$

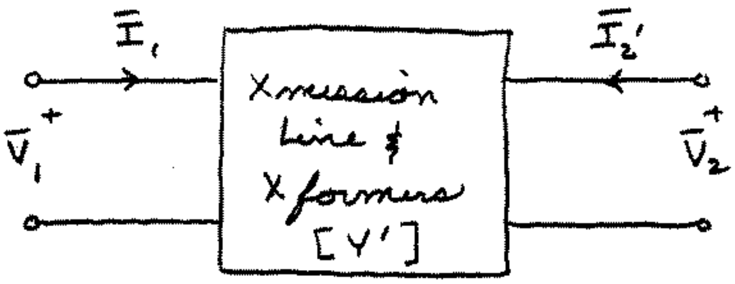
- From Eq (*), write the node equations corresponding to nodes 1 and 2

$$\underline{I} = [A_{11}]\underline{V} + [A_{12}]\underline{V}' \qquad (***)$$

- Eliminate $\underline{V}'$ from (***) by substituting (**) for $\underline{V}'$

$$\begin{aligned} \underline{I} &= [A_{11}]\underline{V} + [A_{12}](-[A_{22}]^{-1}[A_{21}]\underline{V}) \\ &= ([A_{11}] - [A_{12}][A_{22}]^{-1}[A_{21}])\underline{V} = [Y']\underline{V} \end{aligned}$$

Equivalent two-port representation of transmission line and transformer



We can solve the equation on the bottom of the previous page for $\bar{V}_1$ and $\bar{V}_2$. Once these are found, then $\bar{V}_3$ and $\bar{V}_4$ can be determined.

Example

Let $\bar{Z}_1 = \bar{Z}_3 = \bar{Z}_5 = j0.1 \quad \therefore \quad \bar{Y}_1 = \bar{Y}_3 = \bar{Y}_5 = -j10$

$\bar{Y}_2 = \bar{Y}_4 = j0.01$

then

$$\begin{array}{cc} & \begin{array}{cc} A_{11} & A_{12} \end{array} \\ [Y] = & \left[\begin{array}{cc|cc} -j10 & 0 & j10 & 0 \\ 0 & -j10 & 0 & j10 \\ \hline j10 & 0 & -j19.99 & j10 \\ 0 & j10 & j10 & -j19.99 \end{array}\right] \\ & \begin{array}{cc} A_{21} & A_{22} \end{array} \end{array}$$

$$[Y'] = [A_{11}] - [A_{12}][A_{22}]^{-1}[A_{21}]$$

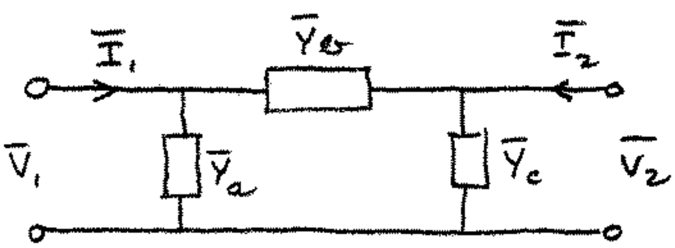
$$= -j10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - j10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -j19.99 & j10 \\ j10 & -j19.99 \end{bmatrix}^{-1}(j10)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2×2 Identity Matrix

$$= -j10\,[I] - (j10)\,[I]\,\frac{\begin{bmatrix} -j19.99 & -j10 \\ -j10 & -j19.99 \end{bmatrix}}{-299.6}\,(j10)\,[I]$$

$$[Y'] = \begin{bmatrix} -j3.328 & j3.338 \\ j3.338 & -j3.328 \end{bmatrix}$$

Let the following π network be used to realize the admittance matrix $[Y']$



Solve for $\bar{Y}_a$, $\bar{Y}_b$ & $\bar{Y}_c$ in the diagram above

$$\left.\begin{aligned} \bar{I}_1 &= \bar{Y}_a \bar{V}_1 + \bar{Y}_b(\bar{V}_1 - \bar{V}_2) \\ \bar{I}_2 &= \bar{Y}_c \bar{V}_2 + \bar{Y}_b(\bar{V}_2 - \bar{V}_1) \end{aligned}\right\} \Rightarrow \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \begin{bmatrix} \bar{Y}_a + \bar{Y}_b & -\bar{Y}_b \\ -\bar{Y}_b & \bar{Y}_b + \bar{Y}_c \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix}$$

$\therefore \quad \bar{Y}_b = -j3.338 \Rightarrow \bar{Z}_b = j0.2996$ (inductive)

$\bar{Y}_a + \bar{Y}_b = -j3.328 \Rightarrow \bar{Y}_a = j0.01 = \bar{Y}_c$ (capacitive)