Basic Principles - Chapter 2

I. Single Phase Circuits - definitions, real & reactive power, sign conventions, power factor

II. Conservation of Complex Power - Tellegan's Theorem, two special Two Port Networks & , difference between a bus and a node

III. Balanced 3ϕ - definition, decoupling of phases, Δ-Y Source & Load transformations

IV. Per Phase Analysis - Balanced 3ϕ Theorem

V. Balanced 3ϕ Power

Other Topics: Phasors, Impedance, Steady State Operation

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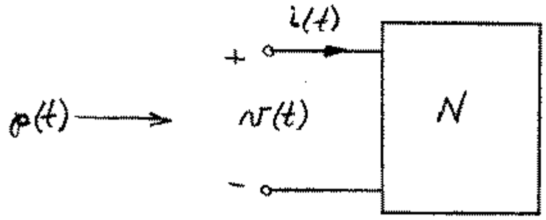
I. Complex Power Supplied to a One Port



Let

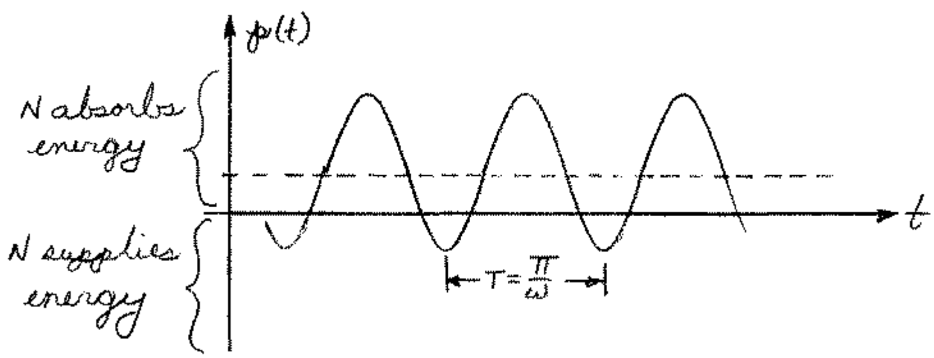
$$v(t) = V_{max} \cos(\omega t + \theta_V) \qquad (2.8)$$

$$i(t) = I_{max} \cos(\omega t + \theta_I) \qquad (2.9)$$

Def. Instantaneous Power Delivered to Network N
(rate at which energy is generated <u>or</u> absorbed by N).

$$p(t) = v(t)\, i(t)$$

$$= \tfrac{1}{2} V_{max} I_{max} \left[\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I) \right] \qquad (2.10)$$



$\omega = 2\pi f = 2\pi 60 Hz$
$= 377$ rad/sec
$T = \frac{1}{120} = 8.33$ msec

Def: Power Factor Angle

$$\phi \triangleq \theta_V - \theta_I \qquad (2.11)$$

Def: Average Power – accomplishes useful work

$$P = \frac{1}{T} \int_0^T p(t)\, dt = \tfrac{1}{2} V_{max} I_{max} \cos\phi \qquad (2.12)$$

Decompose $p(t)$[1]:

$p(t)$ = real work + energy flow in and out of N, but does no real work

[1] Elgerd - Sec 2-3.1 pg 21-27 – use hydralic analogs

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Using trig identities, it can be shown that ($\theta_I = 0$)

$$p(t) = \frac{1}{2} V_{max} I_{max} \left[\cos\phi \, (1 + \cos 2\omega t) - \sin\phi \sin 2\omega t \right]$$

$\cos(\alpha + \beta)$
$= \cos\alpha \cos\beta - \sin\alpha \sin\beta$

Def: Real (or Active or Average) Power

$$P = \frac{1}{T} \int_0^T p(t)\, dt = \frac{1}{2} V_{max} I_{max} \cos\phi$$

Def: Reactive Power

$$Q = \frac{1}{2} V_{max} I_{max} \sin\phi$$

$$\therefore \quad p(t) = P(1 + \cos 2\omega t) - Q \sin 2\omega t$$

$$\left.\begin{array}{l}\text{Sinusoidal Sources} \\ \text{Linear Network}\end{array}\right\} \Longrightarrow \text{phasor notation}$$

Phasor Notation

$$v(t) = V_{max}\cos(\omega t + \theta_V) \iff \bar{V} = \underbrace{\frac{V_{max}}{\sqrt{2}}}_{\text{RMS value of sinusoidal}} e^{j\theta_V} \qquad (2.6)$$

similarly

$$i(t) \iff \bar{I} = \frac{I_{max}}{\sqrt{2}} e^{j\theta_I}$$

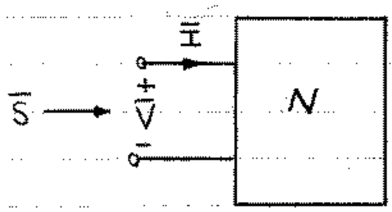
Then

$$P = \tfrac{1}{2} V_{max} I_{max} \cos\phi = |\bar{V}||\bar{I}| \cos\phi$$

$$Q = \tfrac{1}{2} V_{max} I_{max} \sin\phi = |\bar{V}||\bar{I}| \sin\phi$$

ϕ = power factor angle
$= \theta_V - \theta_I$

Def: Complex Power



$$\bar{S} \triangleq P + jQ \qquad (2.18)$$

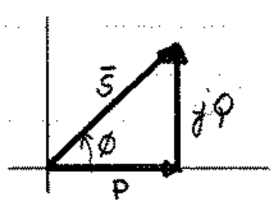
$$= |\bar{V}||\bar{I}|(\cos\phi + j\sin\phi)$$

$$= |\bar{V}||\bar{I}| e^{j\phi}$$

$$= |\bar{V}||\bar{I}| e^{j(\theta_V - \theta_I)}$$

$$= (|\bar{V}| e^{j\theta_V})(|\bar{I}| e^{-j\theta_I})$$

$$= \bar{V}\,\bar{I}^* \qquad (2.17)$$



Note: when expressing sinusoidal quantities as phasors, always express the sinusoidal quantity using the "cosine" function

eg. $v(t) = V_{max}\sin(\omega t + \theta_V) = V_{max}\cos(\omega t + \theta_V - 90°)$

phasor representation of $v(t)$: $\bar{V} = \frac{V_{max}}{\sqrt{2}} e^{j(\theta_V - 90°)}$

	> 0	< 0
P (Watts, W, kW, MW)	N absorbs real power (i.e. load)	N generates real power (i.e. generator)
Q (Volt-ampere reactive: VAr, kKAr, MkAr)	N absorbs reactive power (i.e. inductive load) · I lags V · lagging PF load · $\phi > 0$	N supplies reactive power (i.e. capacitive load) · I leads V · leading PF load · $\phi < 0$

Def: Power Factor $PF \triangleq \cos\phi$

II. Conservation of Complex Power

Theorem of Conservation of Complex Power

$$\sum \begin{matrix}\text{complex power}\\ \text{supplied by}\\ \text{ind sources}\end{matrix} = \sum \begin{matrix}\text{complex power}\\ \text{received by}\\ \text{all other branches}\end{matrix} \iff \sum_{k=1}^{b} \bar{S}_k = 0$$

one port representation

Proof: Use Tellegen's Theorem

Tellegen's Theorem (without proof):

If a set of voltages $v_1'(t), v_2'(t), \ldots, v_b'(t)$ satisfy KVL around all loops in a network; and if a set of currents $i_1''(t), i_2''(t), \ldots i_b''(t)$ satisfy KCL at all nodes in a network, then

$$\sum_{\substack{\text{all}\\ \text{branches}}} v_k'(t)\, i_k''(t) = 0$$

where $v_k'(t)$ and $i_k''(t)$ refer to the voltage across and current thru the k-th branch

Note: $v_k'(t)$ and $i_k''(t)$ do not need to be related via ohm's law!

If we do let v_k' and i_k'' be related by ohm's law, then $v_k' i_k'' = p_k$ and we have

$$\sum_{\substack{\text{all}\\ \text{branches}}} p_k(t) = 0$$

"Summation of instantaneous power $p(t)$ for b branches in the network must be zero"

Other Uses of Tellegen's Theorem

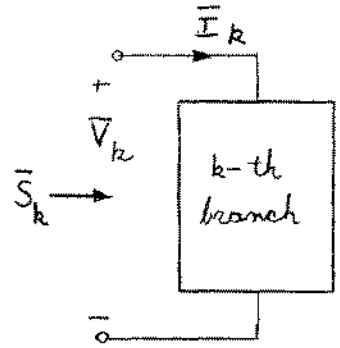
1) Network Sensitivity Calculations - Adjoint Sensitivity Method

Tellegen's Theorem can be extended to phasors (sinusoidal s.s.)

KVL is valid for $\bar{V}$ as well as $v(t)$

KCL " " " $\bar{I}$ & $\bar{I}^*$ " " " $i(t)$

Then
$$\sum_{k=1}^{b} v_k(t)\, i_k(t) = 0 \iff \sum_{k=1}^{b} \bar{V}_k \bar{I}_k^* = 0$$

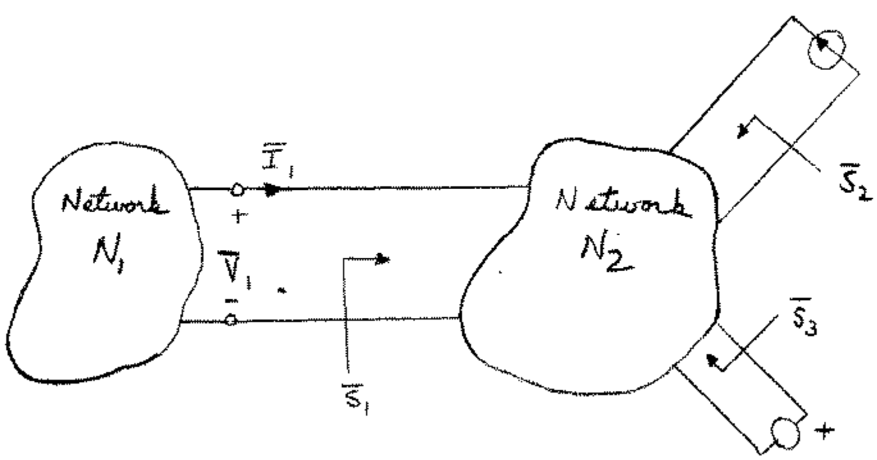


$$\sum_{k=1}^{b} \bar{S}_k = 0$$

Note:

Complex Power is NOT summed into a node.

Bus - a common connecting point for two or more 3ϕ elements
(Kusic page 36)



$\bar{S}_1 + \bar{S}_2 + \bar{S}_3$

$= \sum \bar{S}_i$

i = elements in N_2

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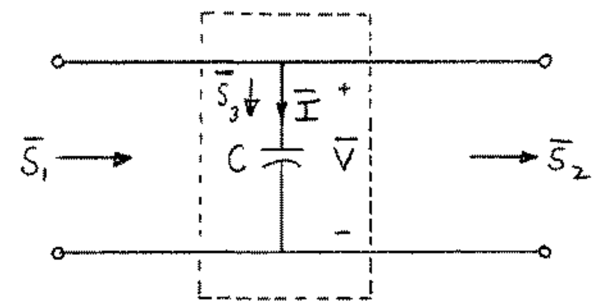
Example 2.4



Find $\bar{S}_2$ in terms of $\bar{S}_1$, C, $\bar{V}$

Sol$^{\underline{n}}$

- theorem of Conservation of Complex Power $\quad \bar{S}_2 = \bar{S}_1 - \bar{S}_3$
- $\bar{S}_3 = \bar{V}\,\bar{I}^*$

  $= \bar{V}(j\omega C \bar{V})^* = -j\omega C|\bar{V}|^2$
- $\bar{S}_2 = \bar{S}_1 - \bar{S}_3$

  $= \bar{S}_1 + j\omega C|\bar{V}|^2 = P_1 + jQ_1 + j\omega C|\bar{V}|^2$

  $= P_2 + jQ_2$

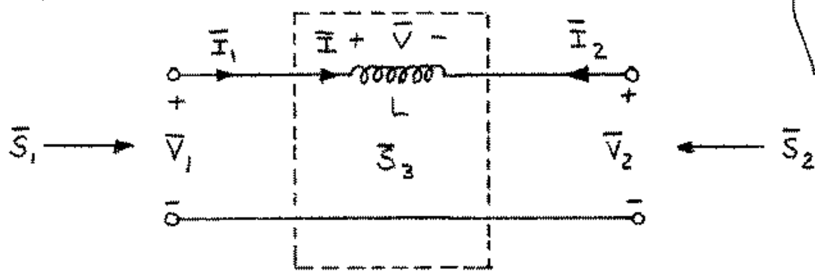
  $P_2 = P_1$

  $Q_2 = Q_1 + \omega C|\bar{V}|^2$
- Note: ① $Q_2 > Q_1$, capacitor acts as a source of reactive power

  ② no gain or loss of real power

  ③ $Q_2 \propto |\bar{V}|^2$

Example 2.5



Let $\bar{S}_1 = P_1 + jQ_1$
then $\bar{S}_2 = -P_1 + jQ_1$

① no loss in real power
② Port 1 & Port 2 each supply exactly $\frac{1}{2}$ of total reactive power absorbed by L

It can be shown using the conservation of complex power that: $\bar{S}_2 = -\bar{S}_1^*$ when $|\bar{V}_2| = |\bar{V}_1|$

Sol^{n}

- Theorem of Conservation of Complex Power: $\bar{S}_1 + \bar{S}_2 = \bar{S}_3$

- $$\bar{S}_3 = \bar{V}\bar{I}^* = (j\omega L\bar{I})\bar{I}^* = j\omega L|\bar{I}|^2$$

- $\bar{S}_1 = \bar{V}_1\bar{I}_1^* = \bar{V}_1\bar{I}^*$

  $\bar{S}_2 = \bar{V}_2\bar{I}_2^* = -\bar{V}_2\bar{I}^*$

- if $|\bar{V}_1| = |\bar{V}_2|$, then $|\bar{S}_1| = |\bar{S}_2| \Rightarrow \begin{array}{l}|P_1| = |P_2| \\ |Q_1| = |Q_2|\end{array}$

- Since $\bar{S}_1 + \bar{S}_2 = \bar{S}_3 = 0 + j\omega L|\bar{I}|^2$

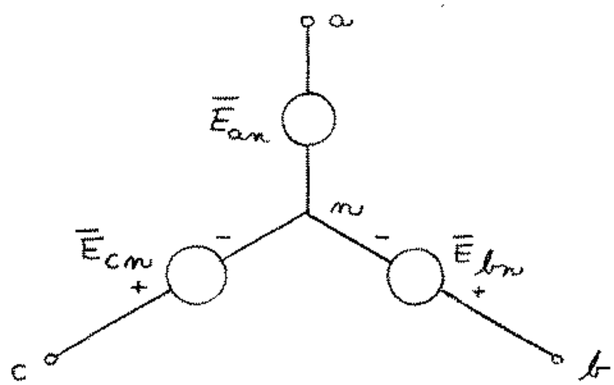
  $P_1 = -P_2$ $\qquad$ $Q_1 + Q_2 = \omega L|\bar{I}|^2$

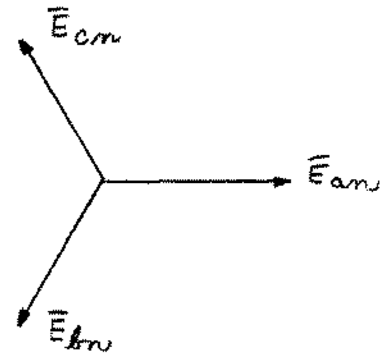
  $\therefore Q_1 = Q_2 = \frac{1}{2}\omega L|\bar{I}|^2$

- $$\left.\begin{array}{l}\bar{S}_1 = P_1 + j\frac{1}{2}\omega L|\bar{I}|^2 \\ \bar{S}_2 = -P_1 + j\frac{1}{2}\omega L|\bar{I}|^2\end{array}\right\} \Rightarrow \bar{S}_2 = -\bar{S}_1^*$$

III. Balanced 3ϕ

Definition
1) Sources: balanced 3ϕ w/ same phase sequence
2) Network: possesses 3ϕ symmetry





$$\bar{E}_{an} = 1\angle 0^\circ \qquad \bar{E}_{an} + \bar{E}_{bn} + \bar{E}_{cn} = 0$$

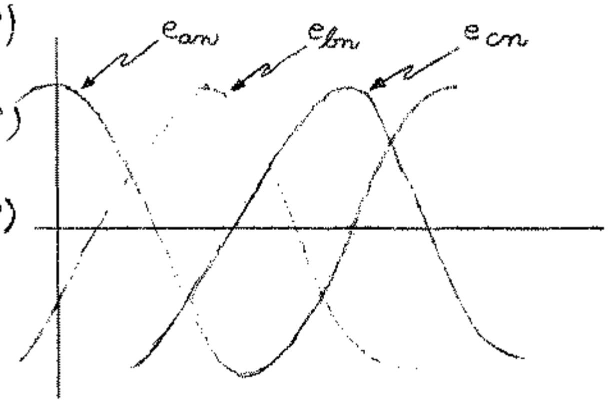
$$\bar{E}_{bn} = 1\angle -120^\circ$$

$$\bar{E}_{cn} = 1\angle -240^\circ$$

$$e_{an}(t) = \sqrt{2}\,|\bar{E}_{an}|\cos(\omega t + 0^\circ)$$

$$e_{bn}(t) = \sqrt{2}\,|\bar{E}_{bn}|\cos(\omega t - 120^\circ)$$

$$e_{cn}(t) = \sqrt{2}\,|\bar{E}_{cn}|\cos(\omega t - 240^\circ)$$



"a" reaches "+" max 1st
"b" " " 2nd
"c" " " 3rd

$$\bar{E}_{ab} = \bar{E}_{an} - \bar{E}_{bn} = \sqrt{3}\,|\bar{E}_{an}|\angle 30^\circ$$

$$\bar{E}_{bc} = \bar{E}_{bn} - \bar{E}_{cn} = \sqrt{3}\,|\bar{E}_{an}|\angle -90^\circ$$

$$\bar{E}_{ca} = \bar{E}_{cn} - \bar{E}_{an} = \sqrt{3}\,|\bar{E}_{an}|\angle -210^\circ$$

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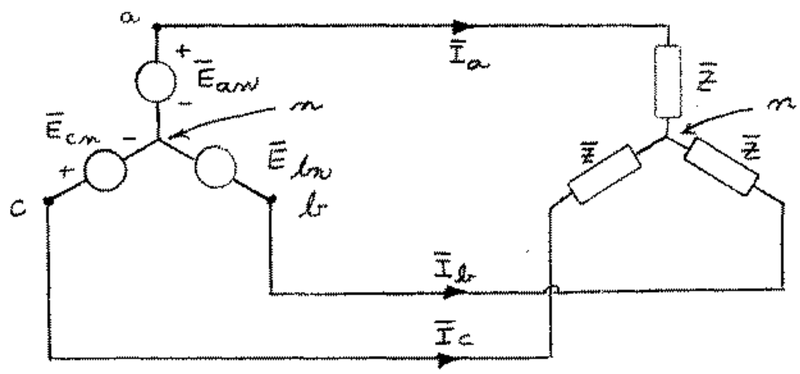
Example 2.9 - Illustrates the concept of "Decoupling of Phases" in Balanced 3ϕ Systems

$\bar{E}_{an}, \bar{E}_{bn}, \bar{E}_{cn}$ are balanced



$\bar{Y} = 1/\bar{Z}$

$$\bar{I}_a = \bar{Y}(\bar{E}_{an} + \bar{V}_{nn'})$$
$$\bar{I}_b = \bar{Y}(\bar{E}_{bn} + \bar{V}_{nn'})$$
$$\bar{I}_c = \bar{Y}(\bar{E}_{cn} + \bar{V}_{nn'})$$

By KCL: $\bar{I}_a + \bar{I}_b + \bar{I}_c = 0$

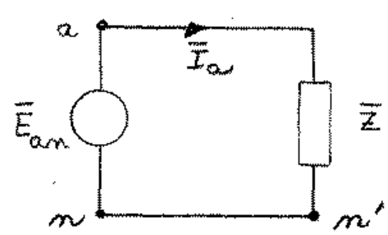
$$\therefore \quad \bar{Y}(\underbrace{\bar{E}_{an} + \bar{E}_{bn} + \bar{E}_{cn}}_{=0,\ \text{since they are balanced}} + 3\bar{V}_{nn'}) = 0$$

$\therefore \quad \bar{V}_{nn'} = 0$

and

$$\left.\begin{aligned} \bar{I}_a &= \bar{Y}\bar{E}_{an} \\ \bar{I}_b &= \bar{Y}\bar{E}_{bn} \\ \bar{I}_c &= \bar{Y}\bar{E}_{cn} \end{aligned}\right\} \text{phases are "decoupled"}$$

Solve 1ϕ circuit, instead of all 3ϕ circuits



This result can be generalized to more complicated balanced 3ϕ systems

Note: Sources and loads must be Y connected for decoupling of phases to occur

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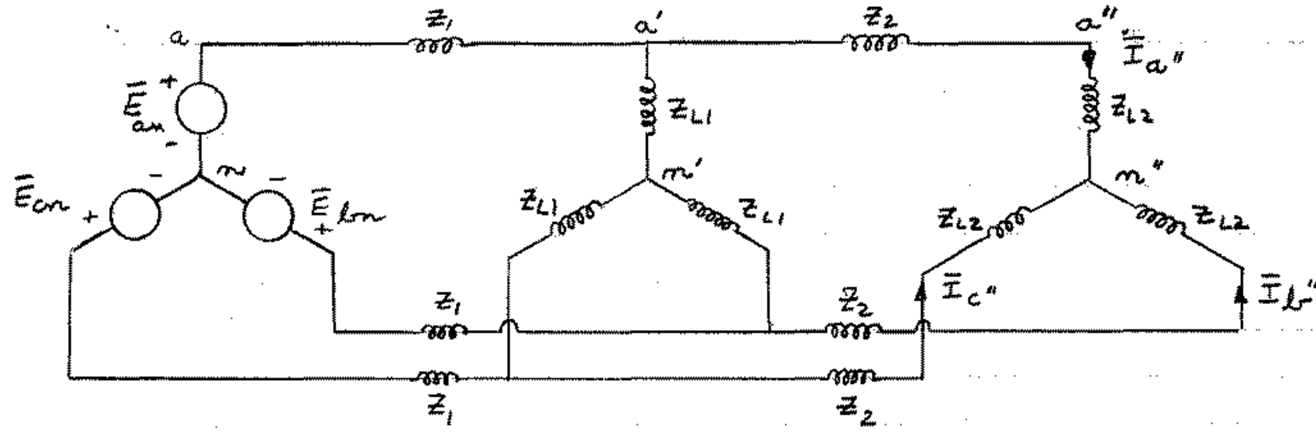
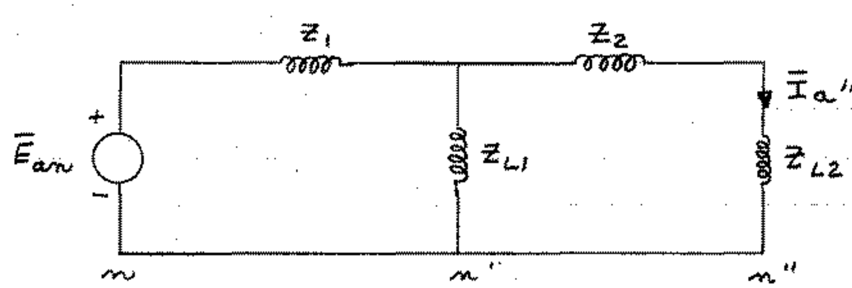
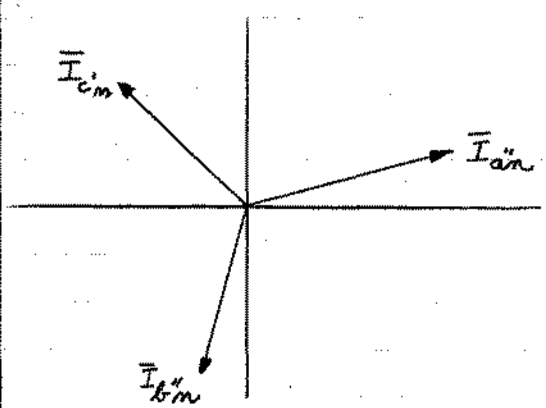


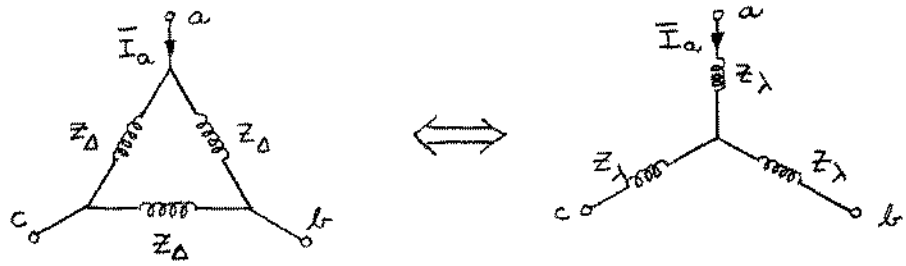




In a balanced 3ϕ system, it is only necessary to analyze one phase. The other 2ϕ are 120° & 240° out of phase

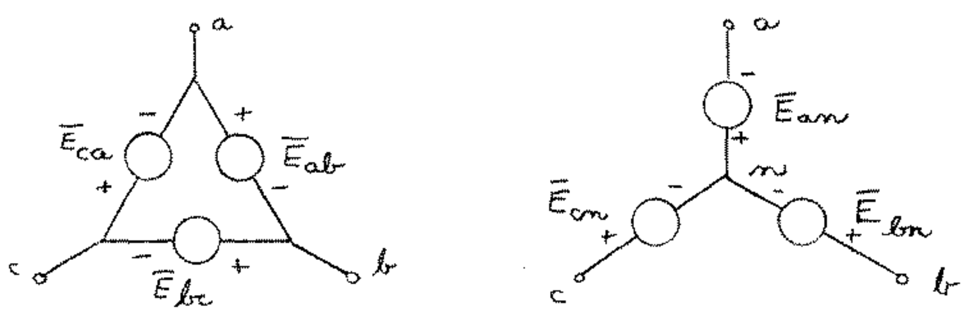
Δ - Y Transformation

Balanced Load



$$Z_\lambda = Z_\Delta / 3$$

Balanced Sources



Positive Sequence

$$\bar{E}_{ab} = \sqrt{3}\, e^{j\pi/6}\, \bar{E}_{an} \tag{2.30}$$

$$\bar{E}_{an} = \frac{1}{\sqrt{3}}\, e^{-j\pi/6}\, \bar{E}_{ab} \tag{2.31}$$

We'll consider "Negative Sequence" quantities when we cover Unbalanced System Operation.

IV. Per Phase Analysis (addresses voltage & current only, not power)

Balanced 3ϕ Theorem

If
1. system is balanced 3ϕ
2. all loads and sources are Y-connected
3. no mutual inductance between phases

Then
1. all neutrals are at the same potential
2. all phases are completely decoupled
3. all corresponding network variables occur in balanced sets of the same sequence as the source

Per Phase Analysis

Given a balanced 3ϕ circuit with no mutual inductance between phases

1. convert all Δ sources & loads into equiv Y connections

2. solve for the desired "phase a" variable (voltage or current) using "phase a" circuit with all neutrals connected together

3. phase b & c variables can be readily calculated
   - subtract 120° from "phase a" to obtain "phase b"
   - " 240° " " " " " " c

4. if necessary, go back to the original circuit to find line-line variables or variables internal to Δ connections

V. Balanced 3 ϕ Power

A. Complex Power

$$\bar{S}_{3\phi} = \bar{V}_{an}\bar{I}_a^* + \bar{V}_{bn}\bar{I}_b^* + \bar{V}_{cn}\bar{I}_c^*$$

$$= \bar{V}_{an}\bar{I}_a^* + \bar{V}_{an}e^{-j120^\circ}\bar{I}_a^* e^{j120^\circ} + \bar{V}_{an}e^{-j240^\circ}\bar{I}_a^* e^{j240^\circ}$$

$$= \bar{V}_{an}\bar{I}_a^* (1+1+1)$$

$$= 3\bar{V}_{an}\bar{I}_a^* \qquad (2.34)$$

B. Real Power

$$p_{3\phi}(t) = v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t)$$

It can be shown (using trig identities) that

$$p_{3\phi}(t) = 3|\bar{V}||\bar{I}|\cos\phi \qquad (2.37)$$

$$= 3P$$

Note that instantaneous power supplied is constant.