3-Phase Converter Modeling for Unbalanced Radial Systems

Karen Miu, Member, IEEE, Chika O. Nwankpa, Member, IEEE,
Dagmar Niebur, Member, IEEE, Razvan Stoicescu, Xiaoguang Yang, Student Member, IEEE

Abstract-- **This paper presents an overview of a power converter model intended for both balanced and unbalanced radial power flow studies [2]. A three-phase, steady-state modeling approach to power converters (*ac/dc – dc/ac*) is outlined where the three-phase rectifier and the three-phase inverter are modeled as three, equivalent, Y-connected single-phase rectifiers and single-phase inverters, respectively. This work specifically focuses on a converter composed of a diode rectifier, a loss-less *dc* link and a pulse-width-modulated inverter is presented. The model can be implemented within a three-phase sequential power flow solver.**

I. INTRODUCTION

Recent developments in power electronics offer the possibility of wide-scale integration of power converters within the supply network [1]. Resulting benefits would include improved control of power delivery, improved power quality for the loads and enhancements to energy management.

Historically, the main power electronics applications in utility systems have been in the transmission sector where HVDC lines, solid state var compensators, unified power flow controllers, and others, have been or are in use to link *ac* systems. As a result, a number of per-phase and/or balanced circuit analysis and power flow solvers were created to handle these devices, [3,4,5,6,7,] and *ac/dc* power systems [9,10,11,12].

Gradually, the use of power electronics is moving into power distribution systems which exhibit unbalanced conditions and single, two and three-phase components and subsystems. For this reason, models for power converters which are now being designed to be able to operate under a level of imbalance are needed, and the previous approaches are not suitable for power distribution systems. Recently in [2], a new, three-phase converter system model (rectifier, *dc* link, inverter) was presented. The model can be used to analyze and develop converter control schemes for both balanced and unbalanced radial power systems. Examples of such systems include: certain terrestrial distribution systems, shipboard power systems, space power and alternative energy source-based systems. It also has applications for other power electronic based equipment such as: Unified Power Controllers (UPC), Dynamic Voltage Regulators (DVAR) and Dynamic-Static Compensators (D-STATCOM).

The results from [2] will be summarized in this work and an approach to modeling 3-phase converters in unbalanced radial systems will be outlined. The main highlights will include:

- the converter modeling approach
- integration within a sequential power flow solver.

Specifically, three-phase converters will be modeled using three, single-phase, grounded, Y-connected converters. The individual phase converters are used to capture the imbalance in the *ac* supply system, yet are designed as a whole to be equivalent to the actual three-phase converters with respect to the average *dc* current through the *dc* link. The model is then integrated into a sequential, unbalanced power flow solver.

II. CONVERTER MODELS

This section presents a modeling approach for three-phase power converters in radial networks shown in Figure 1. We note, since distribution systems are normally operated in a radial manner, the model presented assumes power flows from the rectifier into the *dc* link and then to the inverter.

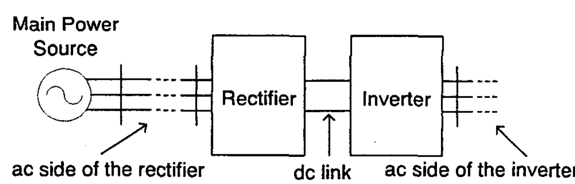

Figure 1. Power Converter Set-up

As such, the rectifiers are assumed to be uncontrolled, diode bridge rectifiers and the inverters are voltage controlled inverters. For bi-directional flow analysis, a controllable rectifier is needed to replace a diode rectifier.

The portion of the network between the main power source and the rectifier will be referred to as the *ac side of the rectifier*, while the portion of the network where average power is leaving the inverter will be referred to as the *ac side of the inverter*.

The three-phase power converter is modeled by combining a (A) rectifier model, (B) *dc*-link and (C) inverter model. The details of each model are now presented.

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R. Stoicescu is with EMA Inc., Philadelphia, PA.

K. Miu, C. Nwankpa, D. Niebur and X. Yang are with the Center for Electric Power Engineering, Drexel University, Philadelphia, PA 19104, USA (e-mail: [Miu, Nwankpa, Niebur]@ece.drexel.edu and xgyang@io.ece.drexel.edu)

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A. Rectifier Model

The approach to model a three-phase full-bridge diode rectifier with three equivalent, grounded Y-connected, single-phase diode rectifiers is now presented. Please see Figure 2.

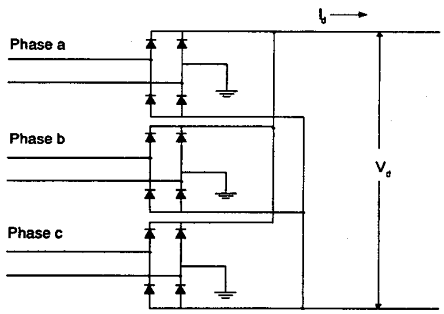

Figure 2. An Equivalent Model Using Three Y-connected Single-Phase Rectifiers

Each of the three, single-phase rectifiers above has the following characteristics: (i) each rectifies the phase voltage for one phase, (ii) their output voltages are in parallel with the *dc* link, (iii) their output currents add and their sum over a period gives the current through the *dc* link. Typically, capacitors sustain the voltage in the *dc* link, and capacitors and inductors are used as filters designed to reduce harmonic content. Therefore, the *dc* voltage, V_d, and average *dc* current, I_d, are assumed to be constant and free of harmonics.

To model a three-phase rectifier using the topology in Fig. 2, a relationship between *ac* and *dc* currents of the actual three-phase rectifier and the model containing three individual rectifiers has to be established. More precisely, the *dc* currents leaving each individual rectifier must be equivalent to the *dc* current that would evolve from one three-phase rectifier. Hence, we first (i) determine the dc currents, then (ii) assess the participation of each phase towards the whole and (iii) make an equivalent of the average *dc* currents. Please see Figure 3 for a flowchart of the modeling process.

1) Determining the DC Current

The *ac* currents entering the respective rectifiers are:

$$i_a(t) = \sqrt{2} I_{ac}^a \sin(\omega t + \varphi_a) \tag{1}$$

$$i_b(t) = \sqrt{2} I_{ac}^b \sin(\omega t + \varphi_b) \tag{2}$$

$$i_c(t) = \sqrt{2} I_{ac}^c \sin(\omega t + \varphi_c) \tag{3}$$

where I_{ac}^a, I_{ac}^b and I_{ac}^c are the RMS values and φ_a, φ_b and φ_c are the phase angles of the *ac* currents with respect to $v_a(t)$. In order to sum the output currents, we choose phase *a* as a reference with reference angle, φ_a, then:

$$\varphi_{ab} = \varphi_a - \varphi_b \tag{4}$$

$$\varphi_{ac} = \varphi_a - \varphi_c \tag{5}$$

To determine the average *dc* current leaving each single-phase rectifier (I_d^a, I_d^b and I_d^c), we integrate the input currents over one and the same period used to determine the RMS values on the *ac* side of the rectifier, here 0 to 2π, for phase *a* reference. For a visual interpretation, please see Figure 3.

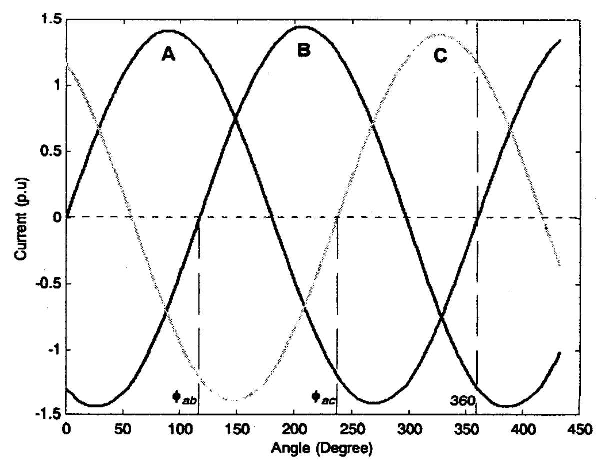

Figure 4. Waveforms of the Three-Phase AC Current in the Three Y-connected Single-Phase Rectifiers

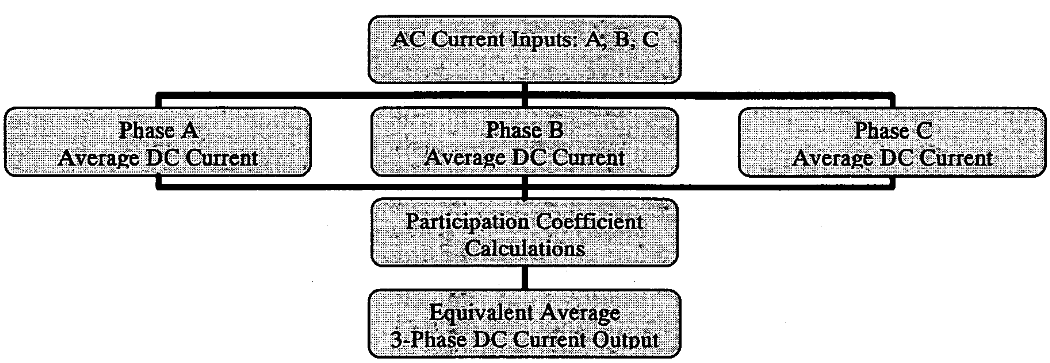

Figure 3. A Flow Chart of the Rectifier Modeling Approach with Respect to Equivalencing the Average *dc* Current Through the *dc* Link When Calculated Using Three, Single-Phase Y-Connected Rectifiers to the Average *dc* Current from a Model of One, Three-Phase Rectifier

The current through the dc link is the sum of the currents entering the rectifiers over one period. Thus, the average value of the current through the dc link for the Y-connected model, $I_d^{3\_Y}$ can be expressed as [2]:

$$I_d^{3\_Y} = I_d^a + I_d^b + I_d^c = 0.9\left(I_{ac}^a + I_{ac}^b + I_{ac}^c\right) \quad (9)$$

Note, for an unbalanced three-phase network I_{ac}^a, I_{ac}^b and I_{ac}^c are not equal. Thus, the contributions of each rectifier are not equal. Following [2], participation coefficients, λ_a, λ_b and λ_c, can be used to make equivalent the sum of the average dc currents for the three Y-connected rectifiers with the average dc current of a three-phase rectifier.

2) Determining Participation Coefficients

Participation coefficients are calculated based on the network imbalance on the ac side of the rectifier. The participation coefficient for phase a is taken as the ratio of I_d^a to the sum of the currents leaving each rectifier:

$$\lambda_a = \frac{I_d^a}{I_d^a + I_d^b + I_d^c} = \frac{I_{ac}^a}{I_{ac}^a + I_{ac}^b + I_{ac}^c} \quad (11)$$

λ_b and λ_c follow in a similar manner. We now investigate the relationship between the sum of the average dc currents from our three, single-phase Y-connected rectifier model and the average dc current from a three-phase rectifier.

3) Equivalencing the DC Current from the Model with that of a Three-Phase Rectifier

If the three-phase rectifier is a three-phase diode bridge rectifier, the average value of the current through the dc link, $I_d^{3-phase}$, can be obtained by integrating it over a period, which is $\pi/3$. Three-phase rectifiers typically assume balanced inputs, thus with balanced conditions in the three-phase ac circuit:

$$I_d^{3-phase} = \frac{1}{\frac{\pi}{3}} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sqrt{2} I_{ac}^a \cos(\omega t) d(\omega t) = 1.35 I_{ac}^a \quad (16)$$

An equivalence coefficient K is defined to resolve the values of average dc current from (9) and (15). K is the ratio of the average value of the dc current when the rectifier is modeled as a three-phase rectifier *to* the dc current when the rectifier is modeled as three, single-phase Y-connected rectifiers:

$$K = \frac{I_d^{3-phase}}{I_d^{3\_Y}} = \frac{1.5}{1 + \frac{\lambda_b}{\lambda_a} + \frac{\lambda_c}{\lambda_a}} \quad (17)$$

Note, K must be determined based on the specific type of rectifier used.

B. DC-Link Model

Under the assumption that the harmonics injected by the rectifier and the inverter can be neglected, the dc link is modeled as a pure dc circuit. The real power P_d transferred through a lossless dc link is obtained as the product of the dc voltage V_d and the average value of the current through the dc link, $I_d^{3-phase}$:

$$P_d = V_d \times I_d^{3-phase} \quad (19)$$

C. Inverter Model

A similar approach to modeling a three-phase inverter as three, equivalent, grounded Y-connected, single-phase voltage source inverters is taken and a connection diagram is shown in Figure 3. Each of the three inverters is a grounded, single-phase, pulse-width-modulated inverter. Their common input is the voltage in the dc link V_d, and their output will be a phase-to-ground voltage for phases a, b and c. By selecting proper switching schemes, these three output voltages can be controlled and the ac bus to which the inverter is connected is modeled as a voltage specified bus.

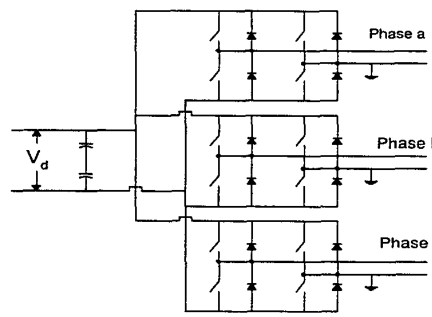

Figure 3. Three Y-connected single-phase inverters

For a single-phase, PWM inverter using unipolar switching [4], with amplitude modulation ratio $m_a < 1$, we obtain:

$$\hat{V}_1 = m_a * V_d \quad (20)$$

where $\hat{V}_1$ is the amplitude of the fundamental voltage waveform. Each equivalent single-phase inverter produces the same amplitude for its subsequent ac phase voltage.

D. AC/DC – DC/AC Converter

When the models for the three-phase rectifier, dc link and the three-phase inverter are combined, the integration limits for the average values of the currents leaving each rectifier on phases a, b and c change. However, since we have assumed the voltage to be constant and harmonic free, we can obtain the contribution from each rectifier by using the equivalance coefficient, K, and the participation coefficients.

Specifically, this converter model is designed for sequential power flow methods. As such, the real power transferred through the converter can be determined from the voltage set point of the inverter and the ac side of the inverter. Consequently, $I_d^{3-phase}$ can be determined. Using (17) and the equivalence coefficient, the equivalent average dc current out of the three, single-phase rectifiers $I_d^{3\text{-}Y}$ can be obtained. Then, using the participation coefficients, λ_a, λ_b and λ_c:

$$I_d^a = \lambda_a I_d^{3\text{-}Y}$$
$$I_d^b = \lambda_b I_d^{3\text{-}Y}$$
$$I_d^c = \lambda_c I_d^{3\text{-}Y}$$

and the RMS currents flowing into the rectifier can be obtained using (6), (7) and (8).

III. POWER FLOW SOLUTION ALGORITHM

A sequential power flow solver was developed in [2] and repeated here. The modeling approach discussed above integrates well with radial power techniques. The following steps outline the power flow procedure:

Step 1. If a converter exists, divide the network into two separate *ac* sub-networks, based on the position of the converter.

Step 2. Solve the *ac* side of the inverter.
Step 2.a. Treat the inverter bus as a voltage-specified bus and solve three-phase power flow.
Step 2.b. Calculate the complex power leaving the inverter bus over all three phases. The total real power transferred through the converter is P_d.

Step 3. Determine the participation coefficients.
Step 3.a. Model the converter and the ac side of the inverter as a lumped three-phase balanced constant power load attached to the rectifier bus using the complex power from *Step 2.b.* Note, reactive power is assumed equal on both sides of the converter only when determining λ_a, λ_b, and λ_c. This assumption is released when solving power flow iterations.
Step 3.b. Apply backward-forward sweeps to the *ac* side of the rectifier resulting in approximate branch currents.
Step 3.c. Use the currents to determine the participation coefficients. Note, these participation coefficients do not change as long as the network topology and the loads power factors do not change.

Step 4. Solve the converter model, starting with the inverter.
Step 4.a. Determine the voltage V_d in the *dc* link using (20).
Step 4.b. Determine I_d using (19).
Step 4.c. Use the equivalence coefficient K to obtain the equivalent average value of the current through the *dc* link using (18).
Step 4.d. Use λ_a, λ_b and λ_c to determine the average *dc* currents leaving the three, single-phase rectifiers.
Step 4.e. Use the rectifier model to determine the magnitude of the *ac* currents entering the three rectifiers, on each phase.

Step 5. Solve the *ac* side of the rectifier.
Step 5.a. Treat the ac rectifier bus as a specified $P|I|$ bus. A balanced real power is determined from Step 2.b. and the individual single-phase rectifier current magnitudes from *Step 4.e* are treated as constant current magnitudes on the rectifier bus. Note, for the power flow analysis, the reactive power injected at the ac rectifier bus is not specified.
Step 5.b. Solve three-phase *ac* power flow for the network on the rectifier side.

IV. COMMENTS AND CONCLUSIONS

An approach for modeling three-phase power converters as three, single-phase Y-connected rectifiers and inverters are advocated as a viable approach to handle unbalanced, radial power systems which contain converters. With the strong interest and plans to embed power converters within actual power distribution systems, the resulting models can assist the development of advanced control schemes for power electronic devices that may have to operate under unbalanced conditions.

The model and approach is dependent on the type of rectifiers and inverters that are used. In this summary, an equivalence coefficient is derived for full-bridge diode rectifiers. This equivalence coefficient relates the three single-phase converters to an actual three-phase converter. The model was successfully implemented into a sequential, radial three-phase power flow solver and simulation results on a 15-bus test system can be found in detail in [2].

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VI. BIOGRAPHIES

Karen Miu received the B.S. M.S. and Ph.D. degrees in electrical engineering from Cornell University, Ithaca NY. She is currently an assistant professor in the Electrical and Computer Engineering Department, Drexel University. Her research interests include distribution system analysis, distribution automation and optimization techniques applied to power systems. She is a recipient of the 2000 NSF Career award and the 2001 ONR Young Investigator Award.

Chika O. Nwankpa (S'88-M'90) received the Magistr Diploma in electric power systems from Leningrad Polytechnical Institute, USSR, in 1986, and the Ph.D. degree in the electrical and computer engineering from the Illinois Institute of Technology, Chicago, in 1990. He is currently an Associate Professor of Electrical and Computer Engineering at Drexel University, Philadelphia, PA. His research interests are in the areas of power systems and power electronics. Dr. Nwankpa is a recipient of the 1994 Presidential Faculty Fellow.

Dagmar Niebur (M '88) received her Diploma in Mathematics and Physics from the University of Dortmund, Germany in 1984. She received her Diploma in Computer Science (1987) and her Ph.D. in Electrical Engineering (1994) from the Swiss Federal Institute of Technology, Lausanne, Switzerland. In 1997 Dagmar Niebur joined Drexel University's Electrical and Computer Engineering Department as an assistant professor. Prior to her engagement at Drexel she held research positions at the Jet Propulsion Laboratory and the Swiss Federal Institute of Technology and a computer engineering position at the University of Lausanne. Her research focuses on intelligent information processing techniques for power system monitoring and control. Dagmar Niebur is a recipient of the 2000 NSF CAREER award.

Razvan Stoicescu received the B. S. degree in electrical engineering from the Politehnica University of Bucharest, Romania in 1995. He received the M.S. degree in electrical engineering from Drexel University in 2000, where he received a Dean's Fellowship Award. He is currently a control systems engineering at EMA Inc. in Philadelphia. He has also held positions at ASA, Co. in Bucharest, Romania.

Xiaoguang Yang received his B.S. degree and M.S. degree from the Electrical Engineering Department, Xi'an Jiaotong University, Xi'an, P.R. China, in 1994 and 1997, respectively. He received his second M.S. degree from the Electrical and Electronic Engineering Department, Nanyang Technological University, Singapore, in 1999. He is currently pursuing a Ph.D. in the Electrical and Computer Engineering Department, Drexel University, Philadelphia, PA.