Static Generator Model for Analog Power Flow Computation

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***Abstract*—Accurate analog models of power system components are required in order to realize an analog computation engine for power systems. Analog computation is an area of continued interest and has certain advantages over traditional digital computation. Among the advantages are physically realizable solutions and significantly faster computation times. This paper focuses on the development of an algebraic static generator model designed for a previously proposed method of analog power flow computation. Prior research in this field has focused on modeling generators dynamically without VAR limitations. The static model proposed here can yield steady state results faster than the prior dynamic models and provides added functionality by incorporating VAR limitations. The model is constructed and verified via analog behavior modeling in PSpice software.**

I. INTRODUCTION

Analog computation of power systems is a continuing field of research[1, 2]. Among the advantages over traditional digital methods are physically realizable solutions and faster computation times. In order to consummate this analog method as a viable tool in power system analysis accurate models of power system components are required. This paper presents a static generator model designed for a specific analog computation method.

Currently power flow computation for large power systems is time intensive. The calculations are non-linear in nature and lengthy iteration schemes are the currently preferred solution. This presents a problem as many assumptions and simplifications are required to solve the equations in a timely manner. In addition, the expansion of the power grid, increasing necessity and complexity of contingency studies and introduction of economic analysis are demanding further computational burden. Traditional digital methods are too slow to solve the aforementioned demands quickly. This affects the security, reliability and market operation of power systems. Ideally a real-time computation tool is preferable, specifically in market activities and operation. Analog computation provides a viable alternative to meet this goal.

The main strength of analog computation is speed and parallelism. The power flow solution is obtained almost instantaneously regardless of the number of components in the network with the power flow method presented here. Essentially the solution is obtained as quickly as the system stabilizes. Experimentation has shown the ability to calculate solutions even faster than real time. In prior research simulation time for a two machine system were typically 10^4 times shorter than the real time simulated phenomena[3]. This is following the approach of modeling generator dynamics for the purpose of transient stability evaluation. In equilibrium this dynamic generator model provides steady state solutions to power flow. The generator model proposed in this paper neglects transient behavior and simply provides the steady state solution while adding VAR limit functionality. This can be advantageous in certain applications where generator dynamics are not of concern. In addition, the computation is algebraic in nature and does not require integration or trigonometric functions as in the dynamic model.

The next section gives an overview on the analog emulation scheme followed by a section presenting the details of the static generator model. Finally simulation results in an emulation environment are shown to verify the functionality of the generator model.

II. DC EMULATION METHODOLGY

A DC emulation power flow method has been proposed in [1] and is reviewed here for an understanding of the application for the generator model in this paper. This approach utilizes multiple resistive networks to compute AC power flow in rectangular coordinates with DC voltages and currents. A figure for a three bus system can be seen in [4]. The emulation is based on the following equation solved in rectangular coordinates:

$$\begin{aligned} I = Y \cdot V = I_{\text{Re}} + jI_{\text{Im}} = \\ \left(Y_{\text{Re}}V_{\text{Re}} - Y_{\text{Im}}V_{\text{Im}}\right) \quad \{\text{real current}\} \\ + j\left(Y_{\text{Im}}V_{\text{Re}} + Y_{\text{Im}}V_{\text{Re}}\right) \quad \{\text{imaginary current}\} \end{aligned} \tag{1}$$

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where the subscripts "Re" and "Im" refer to real and imaginary components respectively.

Each of the four current components seen in (1) are represented by a DC voltage dropped across a resistor. This results in four DC resistive networks. The power system in this emulation is broken up into three main components: generators, transmission lines, and loads. The generators are represented as DC voltage sources, the transmission lines as resistive networks whose size is relative to line parameters and the loads sink current from the networks. The generators excite the networks with real and imaginary DC voltage components and the states (voltages and currents) of the resistive networks provide the steady-state AC power flow solution.

This approach accurately models and calculates power flow for a lossy transmission network. A lossy transmission line model consists of reactive and resistive elements. The relationship between line parameters and the emulation networks can be seen in [2]. For lossless transmission lines (no line resistance) the same approach is taken with the omission of two networks. With no real admittance component two of the current components in (1) do not exist. Reconfigurable variable resistance analog circuits have been developed for the purpose of line modeling [4] and dynamic generator models have been developed in [5].

The generator in this analog computation scheme has to supply the appropriate power to the DC networks. For the dynamic approach the generator model maintains a PV behavior. The computation is based on the swing equation and solves for a generator angle by balancing mechanical power input and electrical power output:

$$M\ddot{\delta} + D\dot{\delta} + P_e(\delta) = P_m \tag{2}$$

With the angular solution of (2) the appropriate voltages in rectangular coordinates are applied to the analog emulation networks governed by analog sine and cosine shapers:

$$V_a = |V| \cdot \cos(\delta) \tag{3}$$

$$V_r = |V| \cdot \sin(\delta) \tag{4}$$

where subscript "a" refers to active and "r" refers to reactive components respectively.

The swing equation is solved in time and will oscillate until an equilibrium point is found (steady-state). A disadvantage of this model is the time it takes to find the steady-state solution. The next section details an algebraic PV generator model for use in this DC emulation technique which finds the steady state solution instantaneously. In addition, this static model incorporates VAR output limitations which prior dynamic models have neglected.

III. ANALOG STATIC GENERATOR MODEL

The static generator model is based on algebraic constraints necessary to maintain the appropriate behavior at the generator bus in the DC emulation scheme. This is separated into two distinct states, PV bus behavior or PQ bus behavior. The latter is utilized when a VAR limit has been reached and clamps the generator output to the specified limitations of the generator. The behavior is governed by:

$$\left.\begin{array}{lll} Q_{\min} \le Q \le Q_{\max} & \Rightarrow & \text{PV bus} \\ \text{otherwise} & \Rightarrow & \text{PQ bus where:} \\ & & Q = Q_{\max} \text{ or } Q = Q_{\min} \end{array}\right\} \tag{5}$$

The user inputs to the model, which could be controlled in analog hardware as voltage inputs, are the specified power output (P), the specified generator voltage magnitude (V) and VAR limits ($Q_{\min}$, $Q_{\max}$). The outputs of the model are the generator voltages in rectangular coordinates. For a PV bus equations (6) and (7) must hold true and for a PQ bus equations (6) and (8) must hold true.

$$P_k = V_{ak} I_{ak} + V_{rk} I_{rk} \tag{6}$$

$$V_k^2 = V_{ak}^2 + V_{rk}^2 \tag{7}$$

$$Q_k = V_{rk} I_{ak} - V_{ak} I_{rk} \tag{8}$$

where subscript "a" refers to active component, "r" refers to reactive and "k" refers to bus number. P_k and V_k are given PV quantities and Q_k the specified VAR limit for bus k.

Equation (6) defines the relationship of the generator voltage and current outputs to the specified power output and equation (7) defines the relationship between the voltage output of the generator and the specified voltage magnitude. From (6) and (7) there are two known variables (P_k, V_k), two unknown variables (V_{ak}, V_{rk}) and two currents which need to be measured in the analog hardware (I_{ak}, I_{rk}). Solving equations (6) and (7) in terms of V_{ak} yields the following polynomial:

$$\left[1 + \left(\frac{I_{ak}}{I_{rk}}\right)^2\right] V_{ak}^2 - \left[2\frac{I_{ak} P_k}{I_{rk}^2}\right] V_{ak} + \left[\left(\frac{P_k}{I_{rk}}\right)^2 - V_k^2\right] = 0 \tag{9}$$

Solving (9) for V_{ak} provides two solutions (V_{ak1}, V_{ak2}) which can be put into a standard form:

$$V_{ak1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{10}$$

where:

$$a = 1 + \left(\frac{I_{ak}}{I_{rk}}\right)^2, \; b = -\frac{2 I_{ak} P_k}{I_{rk}^2}, \; c = \left(\frac{P_k}{I_{rk}}\right)^2 - V_k^2 \tag{11}$$

With V_{ak} known the reactive voltage can be solved for by (12). The sign of V_{rk} is determined by δ and (4).

$$V_{rk} = \pm\sqrt{V_k^2 - V_{ak}^2} \tag{12}$$

An issue with this derivation is that equation (10) yields multiple solutions. There is only one real solution to the power system. Essentially one solution will yield the correct generator voltage and the other will be incorrect. If $V_{ak} \geq V_{rk}$, $V_{ak} = V_{ak1}$ and if $V_{ak} < V_{rk}$, $V_{ak} = V_{ak2}$. In terms of δ :

$$V_{ak} = \begin{cases} V_{ak1} = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} & \text{for } -\dfrac{3\pi}{4} \leq \delta \leq \dfrac{\pi}{4} \\ V_{ak2} = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a} & \text{otherwise} \end{cases} \tag{13}$$

In the event that a VAR limitation is reached the generator output will follow a PQ behavior dictated by equations (6) and (8). Solving for the terminal voltages yields:

$$V_{rk} = \frac{P_k I_{rk} + Q_k I_{ak}}{I_{ak}^2 + I_{rk}^2} \tag{14}$$

$$V_{ak} = \frac{P + V_{rk} I_{rk}}{I_{ak}} \tag{15}$$

Equations (12)-(15) comprise the two state analog static generator model. To realize this model in DC emulation these computations will be conducted via analog hardware (adders, multipliers, etc.) with current feedback from the emulation networks. An evaluation is necessary to determine which state the model operates in and could be conducted via comparators and reference signals for VAR limitations and generator angle. Figure 1 shows a block diagram of the static generator model which consists of user inputs, current feedback and voltage outputs to the DC emulation networks.

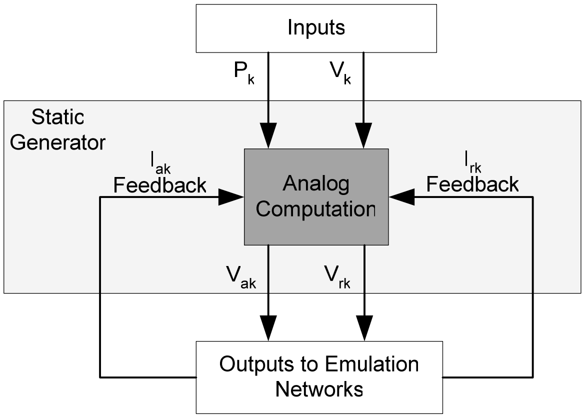

Figure 1. Static Generator Model Block Diagram

IV. SIMULATION RESULTS

Analog behavior modeling (ABM) was utilized in PSpice to model the static generator in DC emulation. ABMs are basically ideal mathematical functions which can be implemented in PSpice and interface with electronic circuit simulations. This approach is suitable for verifying the design and functionality of the computational scheme before designing the analog hardware. The analog hardware can then be designed based on the ABM models.

First a sample three bus power system was constructed in PowerWorld [6] software and power flow results tabulated for benchmarking purposes. Two cases were run. One with VAR limits on the generator and another without. The only difference between the two cases was the limitation of VAR output on the generator. The same three bus system was represented in PSpice utilizing the DC emulation technique. The line impedances were represented by resistors and the slack bus and load bus by voltage sources to match the solution seen in PowerWorld. With this setup the static generator model should yield the same solution (terminal voltage and current output) as the PowerWorld case.

The PSpice schematic of the three bus system is shown in Figure 2. Voltage sources V10 and V11 provide nonzero initial conditions to ensure nonzero initial generator current feedback. Mathematically speaking this is essential to prevent a division by zero in equation (9). At a specified time these sources are switched out and the static generator model is switched into the emulation networks. Figure 3 shows the ABM for equations (12) and (13) which solve for the active and reactive generator voltages for the PV operational mode. Similar modeling was conducted to solve for a, b, and c.

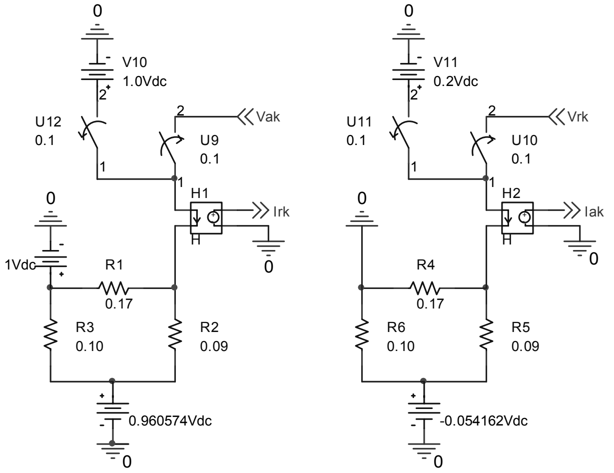

Figure 2. PSpice schematic for DC Emulation Power Flow

Figure 4 shows the ABM for equations (14) and (15) which solve for the active and reactive generator voltages for the PQ operational mode (VAR limit reached and clamped).

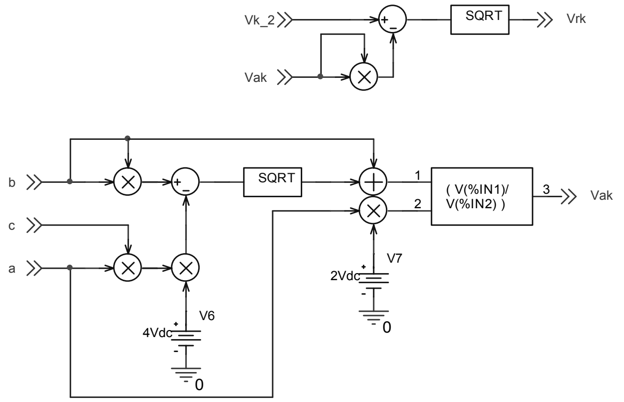

Figure 3. ABM for Static Generator (PV Bus)

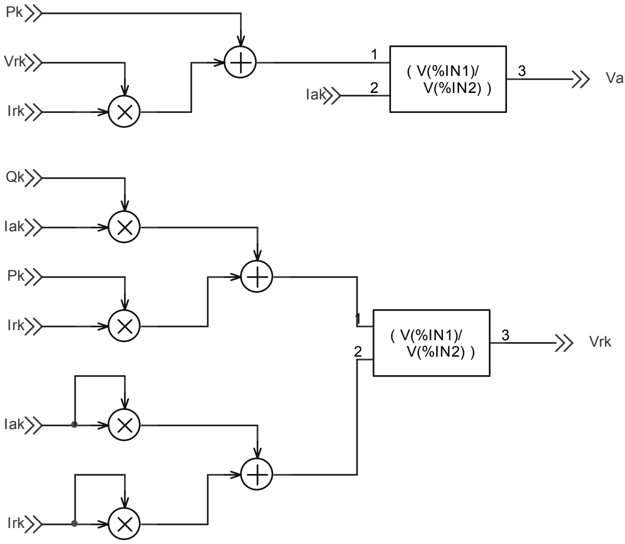

Figure 4. ABM for Static Generator (PQ Bus)

Solutions were obtained for both cases (with and without VAR limit) and compare favorably to the solutions obtained in PowerWorld simulations as shown in Table 1 (no VAR limit) and Table 2 (VAR limit). The results in PowerWorld are in per unit and in PSpice simulation results in volts and amperes. The emulation was configured so that 1 Volt in PSpice is equal to 1 volt per unit and 1 amp corresponds to 1 amp per unit.

TABLE I. COMPARISON OF RESULTS WITHOUT VAR LIMIT

	SIMULATION RESULTS	
	STATIC GENERATOR	***POWERWORLD***
Real Voltage	0.9997	0.9997
Imaginary Voltage	0.0241	0.0241
Real Current	1.0107	1.0107
Imaginary Current	0.4332	0.4331

TABLE II. COMPARISON OF RESULTS WITH VAR LIMIT

	SIMULATION RESULTS	
	STATIC GENERATOR	***POWERWORLD***
Real Voltage	0.9579	0.9669
Imaginary Voltage	0.0255	0.0252
Real Current	1.0420	1.0363
Imaginary Current	-0.0764	-0.0765

V. DISCUSSION

This generator model provides advancement in the DC emulation analog power flow technique by obtaining steady-state solutions faster than prior models and adding VAR limit functionality. With the advent of newer analog fabrication technology such as VLSI it is possible to construct a large scale analog emulator [2]. In this application many of these static generators could be fabricated on a chip in analog hardware with remote configuration and control allowing for multiple power flow cases to be computed extremely quickly. For contingency analysis and power system operation this could be an extremely useful tool with computational speed a few orders of magnitude faster than traditional digital computers.

VI. CONCLUSION

A static generator model for a previously proposed analog power flow technique has been developed theoretically and verified via software simulations. The results compare favorably to a commercially available power flow solver. Implementing this model in analog hardware is left for future work.

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