# Loading Studies for Power Transmission Line Models in the Presence of Non-Fundamental Frequencies

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**Abstract**

This work describes simulations used to investigate commonly used electric power transmission line models. Specifically, the paper focuses on the study of the line model behavior over non-fundamental frequencies and at varying loading conditions. Comparison among the distributed line model and three differently segmented models was made in terms of their performance, characterized through waveform propagation, including attenuation and phase shift.

## 1. INTRODUCTION

Traditional power transmission line models assume uniformly distributed parameters or a lumped parameter configuration (e.g. gamma or pi forms). These models imply constant inductance and capacitance values, independent of frequency. Subsequently, harmonic system analysis tools also adopt lumped parameter models.

Modern power systems exhibit both fundamental and non-fundamental frequency components [1]. With the augmented use of power electronic switches, the level of non-fundamental components present in the network has highly increased. Harmonic components of up to the 15$^{th}$ harmonic for capacitors and the 39$^{th}$ harmonic for power electronic devices are introduced in the network [2]. The presence of these harmonic components is believed to have a strong impact on power transmission line behavior that would require revisiting commonly adopted models. An investigation of this impact would assist in the development of new frequency dependent models.

At Drexel University, an experimental setup and various simulations were developed within the power systems laboratories ([3] and [4]) to investigate the behavior of transmission line models at non-fundamental frequencies and therefore to verify the validity of lumped circuit equivalent models. In [5], voltage and current waveforms for gamma (Γ) line models were investigated, i.e. a lumped equivalent model (one segment gamma model) and finitely segmented models (one to three segment gamma models). The hardware results were compared to state of the art simulation packages. The results showed high sensitivity to load variations.

Therefore, in this paper, a focus on loading studies was taken. In previous work [6], Wilson and Schmidt analyzed the behavior of segmented transmission line models for two loading conditions: open circuit and short circuit. Here, less extreme loading levels will be looked at, including loads above and below the characteristic impedance of the line. Specifically, the line models here studied are loaded with a resistance varying from 100 Ω to 1.1 kΩ. The frequency also varies from 1 Hz to 10 kHz, which is well above the 39$^{th}$ harmonics. The line model performance was observed under different loading conditions and was characterized through wave attenuation and phase shift. The distributed line model embedded in PSpice was also utilized to compare the behavior of distributed and finitely segmented line models.

## 2. DESIGN METHODOLOGY

The main purpose of this work is to investigate frequency characteristics of transmission lines with the goal of validating currently used models. More specifically, in this work, the behavior of differently segmented line models and of the uniformly distributed model is studied under different loading conditions. In order to assess transmission line model behavior, a focus is made with respect to wave attenuation and phase shift. Simulation results for the distributed line model (Figure 1) and for a 5-segment, a 10-segment, and a 20-segment line model are presented in this paper.

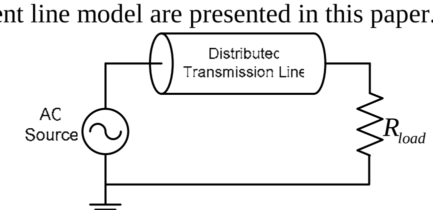


**Figure 1. Distributed Line Model Used in Simulation**

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### 2.1. Development of the Finitely Segmented Model

A 170-mile-long line of Falcon cable [7] was designed and modeled with a gamma (Γ) line model type. See Figure 2 below for the general one-segment Γ model.

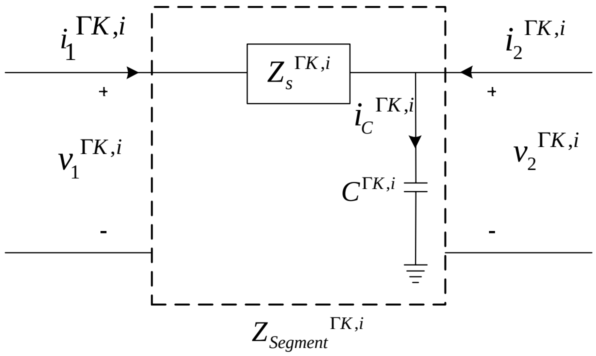


**Figure 2. Notation Definition for Γ-Model Type, 1-Segment Model**

$K$ : total number of segments

$i$ : segment number from receiving end

$v_1^{\Gamma K,i}$ : voltage waveform at the sending-end of the *i*-th segment of stage *K* in the Γ-model type

$i_1^{\Gamma K,i}$ : current waveform through the *i*-th segment of stage *K* in the Γ-model type, seen at the sending-end

$v_2^{\Gamma K,i}$ : voltage waveform at the receiving-end of the *i*-th segment of stage *K* in the Γ-model type

$i_2^{\Gamma K,i}$ : current waveform through the *i*-th segment of stage *K* in the Γ-model type, seen at the receiving-end

$i_C^{\Gamma K,i}$ : current waveform through the shunt capacitor in the *i*-th segment of stage *K* in the Γ-model type, seen at the receiving-end

$Z_S^{\Gamma K,i}$ : 'series' (*S*) impedance of the *i*-th segment of stage *K* in the Γ-model type

$C^{\Gamma K,i}$ : capacitance at the receiving-end side of the *i*-th segment of stage *K* in the Γ-model type

The Falcon cable in delta configuration has the following characteristics defined at a 60 Hz frequency:

$$R_l = 0.0612\ \Omega / mi$$
$$X_l = 0.6081\ \Omega / mi$$
$$X_C = 0.1423\ \text{M}\Omega / mi$$

where $R_l$: line resistance per unit length

$X_l$: line reactance per unit length

$X_C$: reactance of the shunt element per unit length

The shunt capacitance *C* can then be calculated with the following formula:

$$C = \frac{1}{\omega X_C} \cdot \ell = \frac{1}{2\pi f X_C} \cdot \ell$$

where $\ell$: length of the line in miles (receiving end to sending end)

The 5-segment, 10-segment, and 20-segment line models were obtained by uniformly dividing the basic Γ line model into a number of segments *n* (*n* = 5, 10, 20, respectively). Please note that if $n = \infty$ the resulting model is the distributed line model. Figure 3 shows the base model of a lossless finitely segmented line, where *n* is the number of segments.

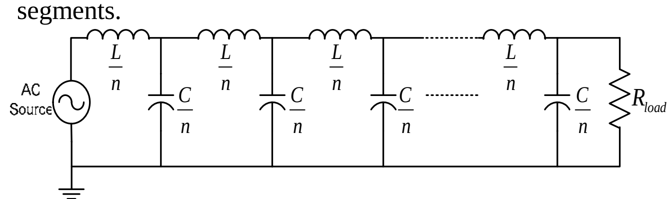


**Figure 3. *N*-Segment Transmission Line Model of a Lossless Line**

Also, please note that the characteristic impedance $Z_C$ of a line is:

$$Z_C = \sqrt{\frac{z}{y}}$$

z = series impedance per unit length per phase

y = shunt admittance per unit length per phase to neutral

Therefore, for this lossless line model:

$$Z_C = \sqrt{L/C} \simeq 295\Omega$$

where:

$$L = \frac{X_l}{2\pi f} = 1.613 \text{ mH/mi}$$

$$C = \frac{1}{2\pi f \cdot X_C} = 18.6408 \text{ nF/mi}$$

## 3. MODEL SIMULATION

### 3.1. Software Simulation Setup

*Cadence PSpice* was utilized to simulate the following:

- A distributed line model of the designed 170-mile-long line. Please note that this is a uniformly distributed model [8]
- A lumped equivalent circuit of 5 Γ segments representing the 170-mile-long line
- A lumped equivalent circuit of 10 Γ segments representing the same line, and
- A lumped equivalent circuit of 20 Γ segments representing the same line

In the simulation results shown in [5], the load resistance was kept constant. Here, loading studies were performed on the line models by varying the load resistance from 100 Ω to 1.1 kΩ.

## 4. RESULTS AND OBSERVATIONS

### 4.1. Voltage Attenuation and Phase Shift as Function of Frequency and Load Resistance

The uniformly distributed line model and the three differently segmented line models (5 segments, 10 segments, and 20 segments) were simulated using *Cadence PSpice*. As frequency $f$ was changed from 1 Hz to 10 kHz and as the load resistance varied from 100 Ω to 1.1 kΩ, the load voltage magnitude and phase angle were monitored to quantify wave attenuation and phase shift. Bode plots of the voltage magnitude and phase were created (Figures 4–12). The voltage attenuation of the distributed line model is shown in Figures 4 and 5, where the difference between the two plots is that Figure 4 is on a logarithmic scale of the frequency, while Figure 5 is not. When looking at the voltage attenuation in the distributed line model, we notice for all frequencies an almost completely flat response of the line loaded with a 300 Ω resistor. This is expected, since this loading level is very close to the characteristic impedance of the line, which results in no wave reflection and therefore very small attenuation. For all other loading levels, the wave behavior follows the voltage function for finite transmission lines:

$$V(x) = I_{Load}\left(Z_{Load}\cosh(\gamma x) + Z_C \sinh(\gamma x)\right)$$

where

- $x$ : distance in miles from the receiving-end of the line. In our case $x = 0$ since we are looking at the receiving-end, i.e. load, voltage
- $I_{Load}$ : current at the load
- $Z_{Load}$ : load impedance
- $\gamma$ : line propagation constant, defined for a lossless line as $\gamma = j\omega\sqrt{LC} = j2\pi\sqrt{LC}$

Please note that voltage as defined above is a function of frequency since the current and the propagation constant both depend on frequency.

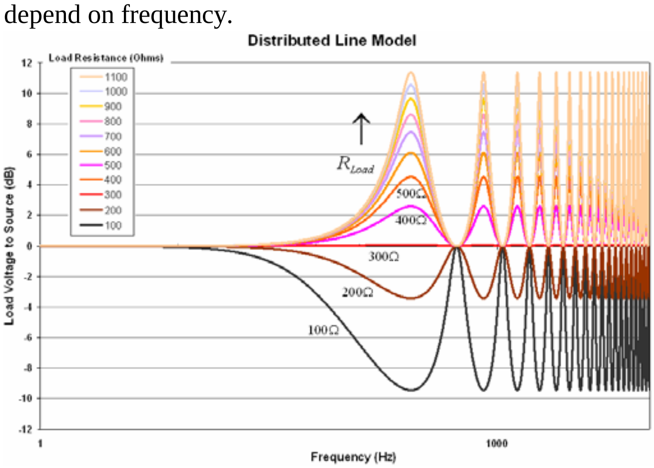


**Figure 4. Voltage Magnitude vs. Frequency for the Distributed Line Model – Logarithmic Scale**

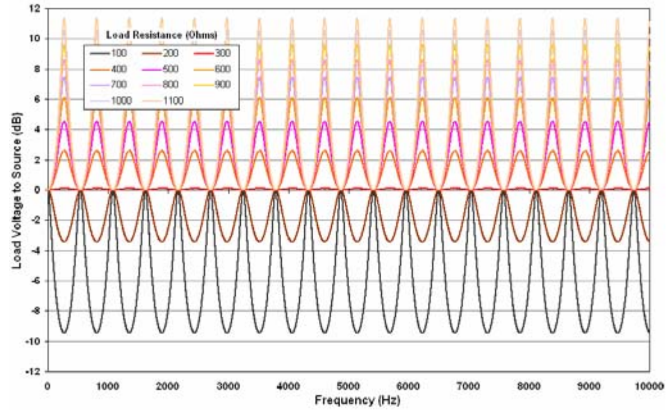


**Figure 5. Voltage Magnitude (dB) vs. Frequency for the Distributed Line Model**

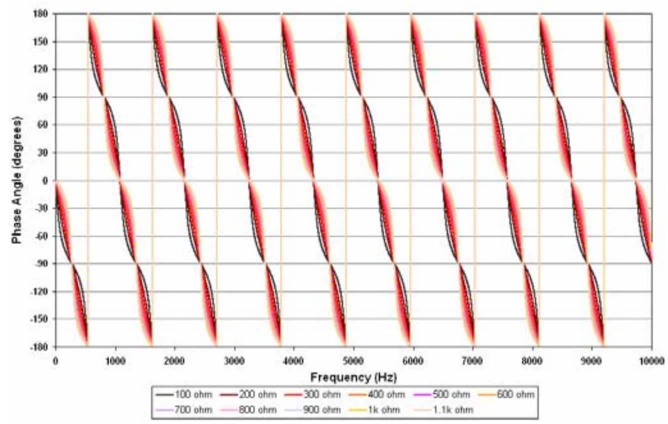


**Figure 6. Load Voltage Phase Angle vs. Frequency for the Distributed Line Mode**

When looking at the voltage phase plot (Figure 6), it can be seen that the load voltage is in phase with the source voltage approximately every 1.1 kHz, which corresponds to those frequencies that make the line length (170 mi) equal to an integer multiple of the wavelength $\lambda$ ($\lambda = c / f$, where $c$ is the speed of light and $f$ the frequency). The frequency values at which the phase shift is 0º, 90º, and 180º seem to remain the same when varying load resistance, while the in-between phase angles vary with load resistance. Also, it is noticed that the frequencies at which the phase shift equals 0º or 180º correspond to the frequencies at which there is no attenuation in load voltage, while those frequencies where the phase shift is 90º correspond to the maximum wave attenuation points.

Figures 7 through 12 show plots of voltage magnitude attenuation and phase shift in the finitely segmented models: the 5-segment, 10-segment and 20-segment. It can be noticed that the behavior of the three finitely segmented line models differs from the distributed line model in the fact that the line attenuation becomes much greater and keeps increasing after a certain frequency, which we can call cut-off frequency, and which seems to increase as the number of segments increases. Also, it can be seen that the model loaded with the 300 Ω resistor is now not as close to the zero attenuation line as with the distributed line model. As the segmentation increases, the line model behaves qualitatively closer to the distributed line model for all frequencies and for all load levels.

Similar observations can be made when looking at the phase plots (Figures 8, 10, and 12). It can be seen that the model behavior matches more closely the distributed line as the segmentation increases. More specifically, it can be seen that the segmented models behave similarly to the distributed model up to a certain cut-off frequency, which increases as the number of segments in the model increases. Also, it can be noticed that the phase shift start settling to either 180º or 0º after the cut-off frequency is reached. As expected for even-number segments (10 and 20) the settling phase shift is 0º, while for odd-number segments (5) is 180º.

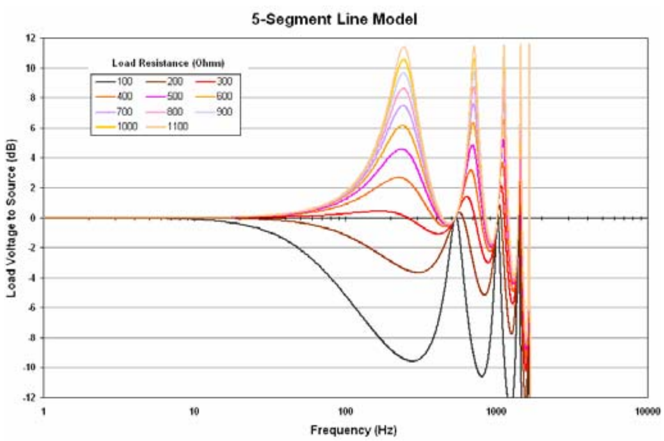

**Figure 7. Bode Plot – Voltage Magnitude vs. Frequency for the 5-Segment Line Model**

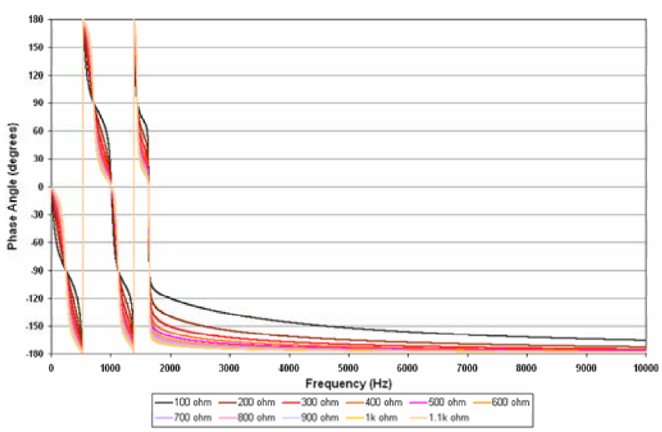

**Figure 8. Load Voltage Phase Angle vs. Frequency for the 5-Segment Line Mode**

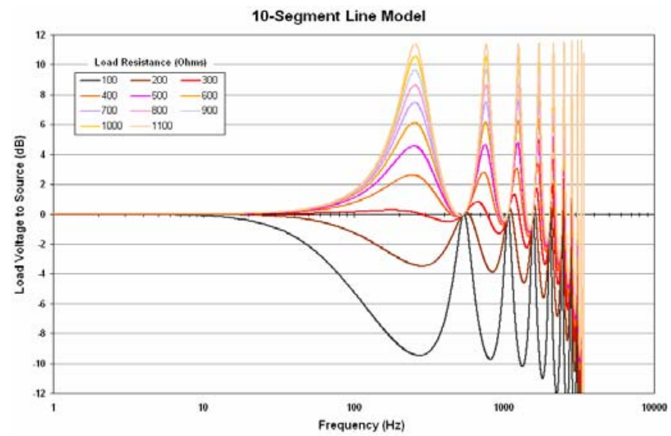

**Figure 9. Bode Plot – Voltage Magnitude vs. Frequency for the 10-Segment Line Model**

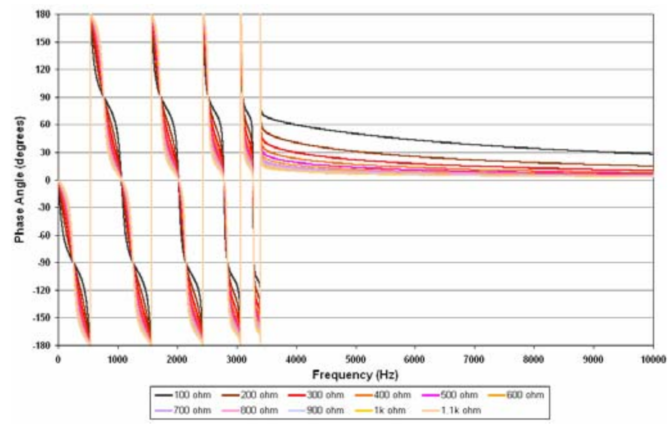

**Figure 10. Load Voltage Phase Angle vs. Frequency for the 10-Segment Line Mode**

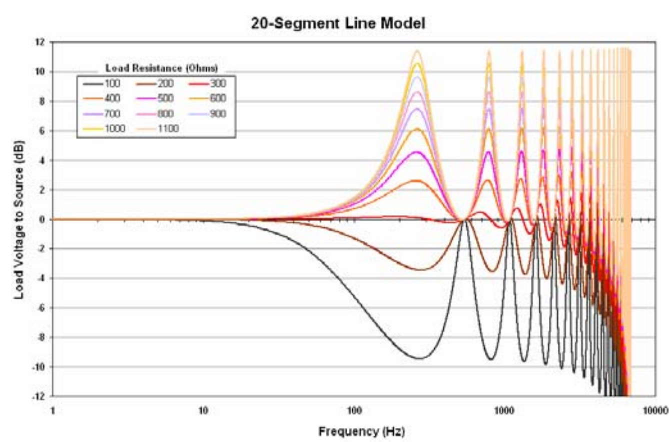

**Figure 11. Bode Plot – Voltage Magnitude vs. Frequency for the 20-Segment Line Model**

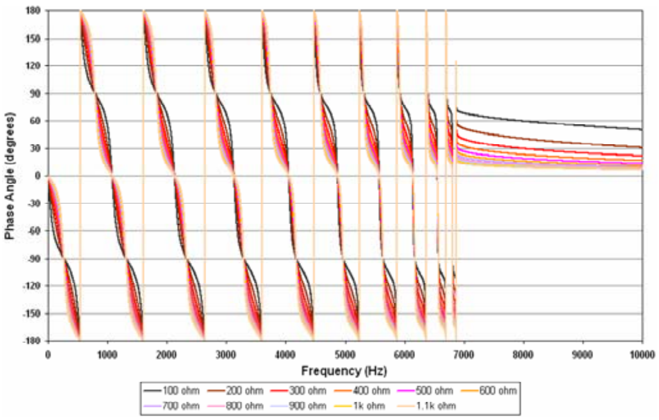


**Figure 12. Load Voltage Phase Angle vs. Frequency for the 20-Segment Line Mode**

## 4.2. Comparison between finitely segmented line models and the distributed line model

In order to quantify the difference that was qualitatively observed in Figures 4 through 12, a comparison of absolute difference in load voltage as a function of frequency was made between each of the segmented line models (the 5-segment, the 10-segment, and the 20-segment) and the distributed line model. Results of this comparison are shown in Figures 13 and 14 for a load resistance of 300 Ω and for a load of 1 kΩ, respectively.

In these plots, it can be clearly seen that the difference in attenuation between the finitely segmented line models and the distributed line model varies with the number of segments of the segmented models. This difference stays within 10 dB up to a frequency of approximately 1.67 kHz for the 5-segment line model, up to 3.3 kHz for the 10-segment model, and up to 6.5 kHz for the 20-segment model. Above the cut-off frequencies, the greater the number of segments of the line model, the faster the model deviates from the distributed model.

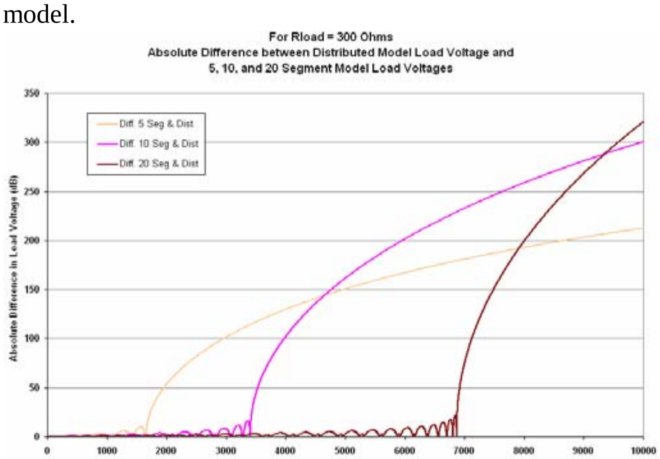


**Figure 13. Absolute difference between voltage attenuation in the distributed line model and in the finitely segmented models, for $R_{load} = 300\ \Omega$**

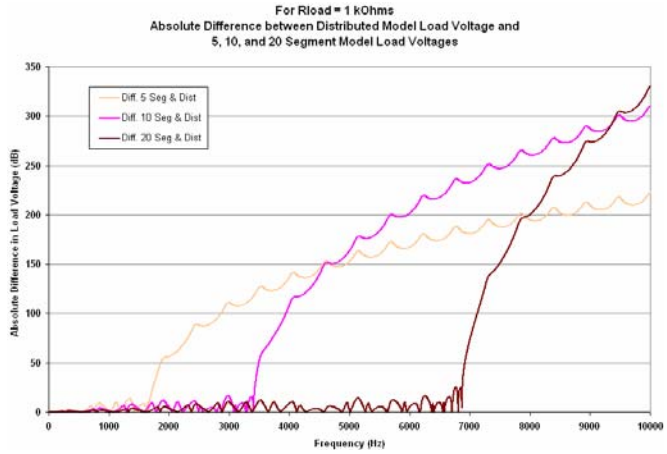


**Figure 14. Absolute difference between voltage attenuation in the distributed line model and in the finitely segmented models, for $R_{load} = 1\ k\Omega$**

It is also noted that these differences between segmented models and the distributed line model increase as the load resistance gets farther from the characteristic impedance of the line (compare Figure 13 to Figure 14 as an example).

## 5. CONCLUSION

The purpose of this work was to investigate traditional transmission line models and their behavior at non-fundamental frequencies and at various loading conditions. More specifically, the distributed line model was compared to lumped, finitely segmented line models at different frequencies, which varied from 1 Hz to 10 kHz, and at different load resistances, from 100 Ω to 1.1 kΩ.

Equivalent circuits were developed to represent a 170-mile long transmission line of Falcon cable. A distributed model and three finitely-segmented gamma-type models of this line were created. Specifically, a 5-segment, a 10-segment and a 20-segment model were utilized to characterize the model behavior with frequency as the segmentation increases. The line model performance was observed under the different loading conditions and was characterized through wave attenuation and phase shift.

From the simulation results, it can be seen that when the line is loaded with resistances above the characteristic impedance of the line an increase in load voltage with respect to the source voltage is observed, while when below the characteristic impedance the voltage is attenuated. The observed behavior follows the voltage equation for finite transmission lines for all frequencies when looking at the distributed model, and up to the cut-off frequency for the finitely segmented models. These cut-off frequencies increase as the number of segments increases. It is also noted that as the segmentation increases, the line model seems to behave qualitatively and quantitatively closer to the distributed line model for all frequencies and for all load levels. In future work, these observations will assist in the development of frequency dependent mathematical models of the transmission line behavior.

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## BIOGRAPHIES

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