Reconfigurable Phase Shifting Transformer for Analog Computation

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***Abstract*—This paper focuses on the development of a phase shifting transformer model designed for a previously proposed method of analog power flow computation. Prior research in this field has modeled generators, loads and transmission lines. Accurate analog models of power system components are required in order to realize an analog computation engine for power systems. Analog computation is an area of continued interest and has certain advantages over traditional digital computation. Among the advantages are physically realizable solutions and significantly faster computation times. The transformer model proposed here provides a more accurate depiction of the network and captures the behavior of phase shifting transformers. The transformer is modeled in analog form. The model is constructed and verified via software simulation.**

***Index Terms*—Analog computers, Reconfigurable emulation, Transformer model.**

I. INTRODUCTION

Analog computation of power systems is a continuing field of research[1-3]. Among the advantages over traditional digital methods are physically realizable solutions and faster computation times. In order to consummate this analog method as a viable tool in power system analysis accurate models of power system components are required. This paper presents a phase shifting transformer model designed for a specific analog computation method.

The phase shifter model developed here is our first experience with a series connected regulated equipment. The work in this paper introduces a transformer model which emulates the following characteristics:

- Remotely reconfigurable parameters
- Phase Shifting
- Load voltage regulation via tap changing

Currently power flow computation for large power systems is time intensive. The calculations are non-linear in nature and lengthy iteration schemes are the currently preferred solution. This presents a problem as many assumptions and simplifications are required to solve the equations in a timely manner. In addition, the expansion of the power grid, increasing necessity and complexity of contingency studies and introduction of economic analysis are demanding further computational burden. Traditional digital methods are slow to solve the aforementioned demands quickly. This affects the security, reliability and market operation of power systems. Ideally a real-time computation tool is preferable, specifically in market activities and operation. Analog computation provides a viable alternative to meet this goal.

The main strength of analog computation is speed and parallelism. The power flow solution is obtained almost instantaneously regardless of the number of components in the network. The solution is obtained as quickly as the system stabilizes. Experimentation has shown the ability to calculate solutions even faster than real time. In prior research simulation time for a two machine system were typically 10^4 times shorter than the real time simulated phenomena[4]. This is following the approach of modeling generator dynamics for the purpose of transient stability evaluation. In equilibrium this dynamic generator model provides steady state solutions to power flow. Prior research has developed static and dynamic generator models[4-6], dynamic load models[7] and static transmission line models[2, 8] for the analog computation scheme utilized here.

The transformer model proposed in this paper enhances the currently existing network model for this computation scheme. An overview on the analog emulation scheme is presented in Section II. Details of the static phase shifting transformer model are presented in Section III. Simulation results are reported in Section IV.

II. DC EMULATION METHODOLOGY

A DC emulation power flow method has been proposed in [2] and is reviewed here for an understanding of the application for the transformer model in this paper. This approach utilizes multiple resistive networks to compute AC power flow in rectangular coordinates with DC voltages and currents. The emulation is based on the following equation solved in rectangular coordinates:

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$$\begin{aligned} I = Y\cdot V &= \left(Y_{\mathrm{Re}}+jY_{\mathrm{Im}}\right)\left(V_{\mathrm{Re}}+jV_{\mathrm{Im}}\right) \\ &= I_{\mathrm{Re}}+jI_{\mathrm{Im}} \\ &= \left(Y_{\mathrm{Re}}V_{\mathrm{Re}}-Y_{\mathrm{Im}}V_{\mathrm{Im}}\right) \qquad \{\text{real current}\} \\ &\quad + j\left(Y_{\mathrm{Im}}V_{\mathrm{Re}}+Y_{\mathrm{Re}}V_{\mathrm{Im}}\right) \quad \{\text{imaginary current}\} \end{aligned} \tag{1}$$

where the subscripts “Re” and “Im” refer to real and imaginary components respectively.

Each of the four current components seen in (1) are represented by a DC voltage dropped across a resistor. This results in four DC resistive networks which represent the AC power system network. The power system in past emulations was broken up into three main components: generators, network and loads. The generators were represented as DC voltage sources, the power system network as resistive networks whose size is relative to the network parameters and the loads sink current from the networks. Note that this network model requires the network to be represented by fixed impedances. System frequency is assumed to be constant. The generators excite the networks with real and imaginary DC voltage components and the states (voltages and currents) of the resistive networks provide the AC power flow solution.

The generator in this analog computation scheme has to supply the appropriate power to the DC networks. The generator was modeled dynamically via the swing equation and maintains a PV behavior[5]. The generator angle is solved for by balancing mechanical power input and electrical power output:

$$M\ddot{\delta}+D\dot{\delta}+P_e\left(\delta\right)=P_m \tag{2}$$

With the angular solution of (2), obtained via analog integrators, the appropriate voltages in rectangular coordinates are applied to the emulation networks governed by analog sine and cosine shapers:

$$V_{\mathrm{Re}}=\left|V\right|\cdot\cos\left(\delta\right) \tag{3}$$

$$V_{\mathrm{Im}}=\left|V\right|\cdot\sin\left(\delta\right) \tag{4}$$

The load is modeled dynamically as a PQ bus interfacing with the DC emulation networks[7]. In this paper the phase shifting transformer model is lumped into the power system network and was developed to function in the framework of this DC emulation scheme.

III. ANALOG TRANSFORMER MODEL

The phase shifting transformer model was developed by deriving the DC emulation representation of the transformer from a circuit representation of the transformer. With the model suitable for DC emulation analog circuits were developed to realize the model which allow for parameter reconfiguration and tap setting control.

The circuit representation of the phase shifting transformer is shown in Fig. 1[10]. It consists of an ideal transformer of turns ratio 1: t and a series impedance. The computation is done in per-unit with a nominal turns ratio of one.

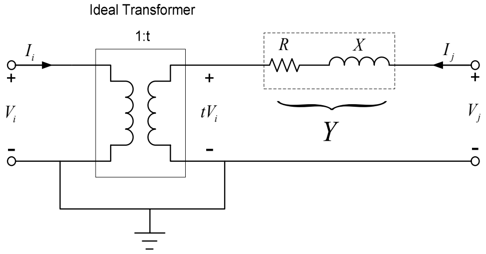

Fig. 1. Circuit Representation of a Transformer

Developing equations for the voltages and currents in the circuit yields the following equation:

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} \left|t\right|^2 Y & -t^* Y \\ -tY & Y \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{5}$$

A pi equivalent circuit can be developed for this transformer from (5) so long as the tap setting *t* does not have a phase shift. A model for a tap changing transformer has been developed in [9]. The pi equivalent circuit of a tap changing transformer is shown in Fig. 2.

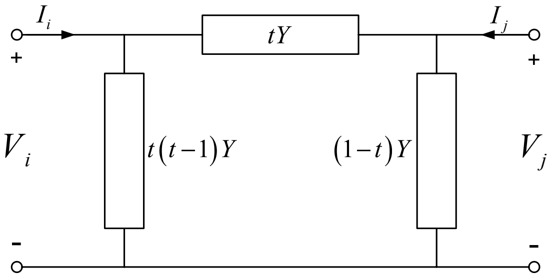

Fig. 2. Equivalent circuit of a Tap Changing Transformer

The pi equivalent circuit shown in Fig. 2 can be written in the Y_{bus} matrix form:

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} \left|t\right|^2 Y & -tY \\ -tY & Y \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{6}$$

If a phase shift is present the off diagonal entries of (5) become unequal and the circuit will no longer be realizable. The modified pi equivalent circuit with a dependent current source shown in Fig. 3 properly models the tap changing transfer behavior with or without phase shifting. The dependent current source compensates for the current error induced into the pi-equivalent circuit when phase shifting is present. I-V characteristics on this model hold as long as the current correction side of the transformer is connected to a load or a generator bus.

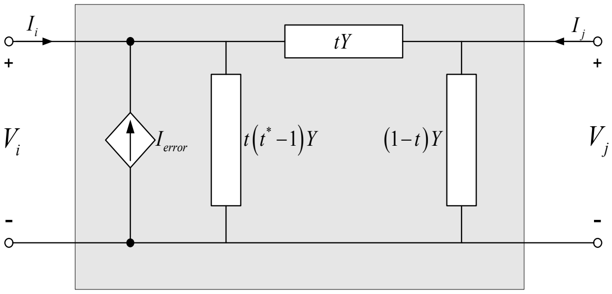

Fig. 3. Equivalent Circuit of Phase Shifting Transformer

The error derived from the pi equivalent circuit in Fig. 2 and (5) is quantified as a function of both the complex tap setting and the voltage V$_j$:

$$I_{error} = \left(t - t^*\right) Y \cdot V_j \tag{7}$$

In order to be used in DC emulation the equivalent circuit for the phase shifting transformer was transformed into rectangular coordinates similar to (1) and is represented in emulation with four DC resistive networks. Representing the phase shift of the transformer by θ the current source I_{error} :

$$\begin{aligned} I_{error} &= \left(t - t^*\right) Y \cdot V_j = \left(j2\left|t\right|\sin(\theta)\right) Y \cdot V_j = \\ &\left(j2t_{\text{Im}}\right) Y \cdot V_j = I_{\text{Re}} + jI_{\text{Im}} = \\ &-\ 2t_{\text{Im}} Y_{\text{Re}} V_{j\,\text{Im}} \quad \text{network 1} \\ &-2t_{\text{Im}} Y_{\text{Im}} V_{j\,\text{Re}} \quad \text{network 2} \\ &-\ j2t_{\text{Im}} Y_{\text{Im}} V_{j\,\text{Im}} \quad \text{network 3} \\ &+\ j2t_{\text{Im}} Y_{\text{Re}} V_{j\,\text{Re}} \quad \text{network 4} \end{aligned} \tag{8}$$

The four DC emulation networks are constructed to emulate the behavior of the phase shifting transformer as shown in Fig. 4. The real and imaginary resistors values for one of the shunt elements in Fig. 3 to be used in the DC networks are sized as follows:

$$\frac{1}{\left[t\left(t^*-1\right)Y\right]} = A + jB \tag{9}$$

$$R_{\text{Re}} = \frac{A^2 + B^2}{A} \tag{10}$$

$$R_{\text{Im}} = \frac{A^2 + B^2}{B} \tag{11}$$

The rest of the element resistors are sized in similar. The real resistor (R$_{\text{Re}}$) is used in networks one and four while the imaginary resistor (R$_{\text{Im}}$) is used for networks two and three. For the current source imaginary voltages are used in networks one and three while real voltages for networks two and four.

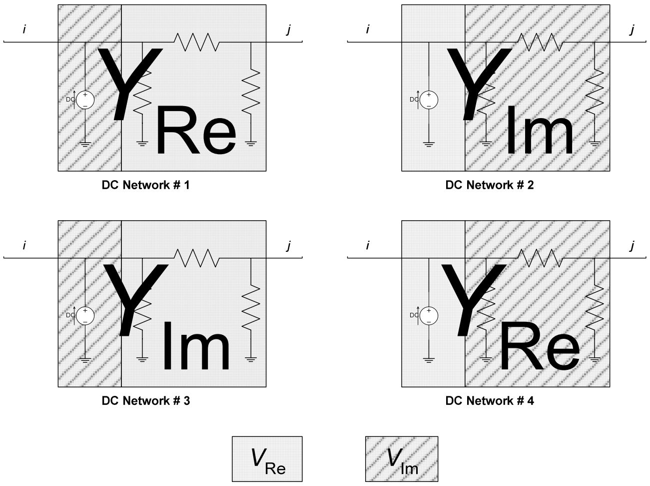

Fig. 4. Representation of the Phase Shifting Transformer with 4 DC networks

Actual use of resistors in DC emulation would require manual intervention to configure and alter the analog system. In order to overcome this problem a remotely reconfigurable Operational Transconductance Amplifier (OTA) based variable resistor was proposed in [8] and used to transfer the DC networks into reconfigurable circuits. The transconductance gain (g_m) of an OTA is controllable through an external source over a wide range, allowing remote reconfigurability. The principal operation of an OTA is a voltage controlled current source (VCCS). Fig. 5 shows an ideal OTA. The amplifier has a differential voltage input and a current output. The current output is related to the differential input through a transconductance gain (g_m) controllable through an external source. Ideally the output would be:

$$i_o = g_m v_{in} \tag{12}$$

where v_{in} is the differential input voltage.

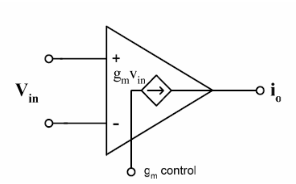

Fig. 5. Ideal OTA

Each component of I_{error} used in the four DC network analysis is realized using an OTA. I_{error} in network one will have a transconductance gain of:

$$g_m = -2t_{\text{Im}} Y_{\text{Re}} \tag{13}$$

and

$$v_{in} = V_{j\,\text{Im}} \tag{14}$$

The OTA based variable resistor shown in Fig. 6 [11] consists of two OTAs to address bidirectional current flow.

The two inputs (V_1, V_2) mimic terminals of a resistor and the i_{abc} is the bias current used to control the resistance of the model similar to the wiper on a potentiometer. The effective resistance seen between terminals V_1 and V_2 is governed by the following equation[8]:

$$R_{eff} = \frac{2R + R_A}{g_m R_A} \tag{15}$$

The sizing of resistors R and R_A along with the range of transconductance gain g_m will determine the behavior of the circuit. The resistors in the transformer model are built utilizing these variable resistor circuits in the manner shown in Fig. 6. The transformer parameters are remotely controllable and reconfigurable via the bias current in the OTA circuits.

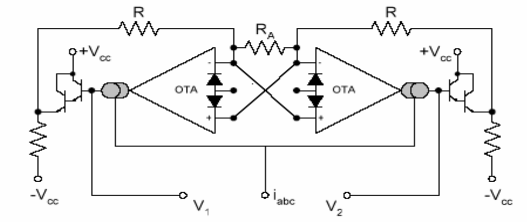

Fig. 6. OTA Based Variable Resistor

IV. SIMULATION RESULTS

Software simulation for the purpose of model validation was performed. Four independent networks were used to simulate a phase shifting transformer in PSpice. The combination of these networks constitutes the complete test system.

To verify the transformer model, results from PSpice simulation are compared to those obtained from a traditional load flow solver such as PowerWorld[12]. Fig. 7 shows the PowerWorld system with load flow results. The phase shifting transformer is connected between bus 3 and bus 4.

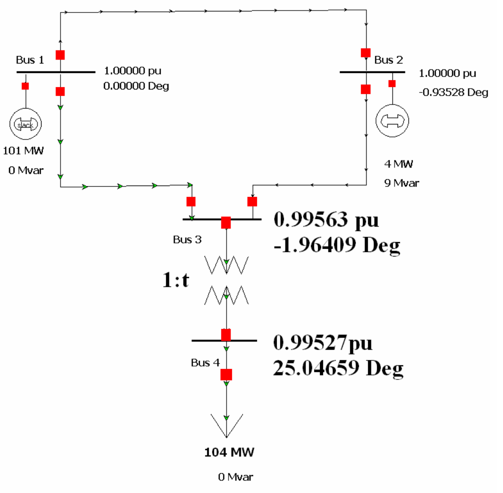

Fig. 7. PowerWorld results for 4 Bus Power System

Based on PowerWorld parameters, PSpice circuits for simulation are built. Fig. 8 shows the PSpice circuit for DC network 1. Currents flowing in the primary and secondary side of the transformer are compared in Table I.

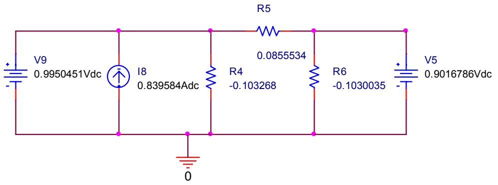

Fig. 8. PSpice schematic circuit for Network 1

TABLE I
CURRENT COMPARISON

	I_i (A)		I_j(A)	
	Re	Im	Re	Im
PowerWorld	1.0476	-0.0908	-0.9467	-0.4423
PSpice	1.0472	-0.0909	-0.9465	-0.4423
% error	0.038	0.11	0.021	0

For DC emulation of the phase shifting transformer, all networks components should be constructed with OTAs which enable the analog system to become reconfigurable. Scaling of system parameters to proper current and voltage level is required to guarantee that the OTA is functioning in the linear operating region. The value for each OTA based variable resistor is dependent on the complex tap position t of the transformer, while the OTA based variable current source is dependant on the shift tap. Therefore, OTA based components value change to regulate the output voltage of the phase shifting transformer. The OTA based analog emulator will produce an error if compared with the result from PowerWorld. This error can be attributed to the non-ideal characteristics of the OTA.

In these simulation results, voltage sources were placed on the side of the phase shifting transformer model that has voltage controlled current sources. This is necessary in the DC emulation method to ensure proper network voltages. All injections to the emulation networks must be modeled as voltage sources. The implementation of this phase shifting transformer model in hardware will replace these current sources with current controlled voltage sources. This has been done when modeling other injection into the emulation network such as generator and load models[6, 7].

V. CONCLUSION

This paper formulates, by means of the equivalent circuit and the proposed DC emulation technique in [2], a model of a phase shifting transformer for analog computation. The model properly models the I-V characteristic of a phase shifting transformer. The analog circuit can be realized via various reconfigurable OTA based components. The OTAs allow remote control and configuration of transformer parameters and tap setting. Simulation results verify the design of the model. The PSpice solution closely matches the desired PowerWorld solution. Current work in implementing the proposed phase shifting transformer model in a transmission network is in development.

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