

Slack Bus Modeling and Cost Analysis of Distributed Generator Installations

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Abstract: The installation and operation of distributed generators (DGs) has great potential for local utilities to improve distribution system reliability and lower their operating and expansion planning costs. To evaluate this potential, distribution system analyses must reflect its new operating environment with significant DG. Resulting tools can be utilized by both utilities and DG owners to improve their decision making algorithms. As such, this work investigates two different slack bus models for unbalanced distribution power flow and their impacts on subsequent cost analysis. The models include the traditional single slack bus model which assigns the substation as the slack bus and a distributed slack bus model which assigns slack to the substation and DGs according to network-based participation factors. Detailed expressions for cost analysis which directly depend on the distributed slack bus model are presented and discussed. Simulations illustrate that the different slack bus models have significant impacts on cost analysis. The difference in estimated annual profit for local utilities reached up to 265% using the different slack models.

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Introduction

The installation of distributed generators (DGs) within distribution systems creates opportunities and challenges for both local utilities and independent power providers (IPPs). Both require tools to assess their own cost and to maximize their profits under competition. In addition, since DGs may belong to different owners and local utilities own and operate the network, a fair pricing scheme for the operation of DGs is important. Therefore, new analysis tools for distribution system planning and operating techniques need to be developed to accommodate the changed operating distribution environment (Ackermann et al. 2001; Barker and DeMello 2000).

The essential tool for distribution system analysis is three-phase distribution power flow. Since the system loss is unknown before power flow calculations, slack bus models are used to absorb the mismatch power (Tong and Miu 2005; Okamura et al. 1975; Calovic and Strezoski 1981). Subsequent cost analyses were also investigated for transmission systems and power markets (Meisel 1993; Zobjan and Ilic 1997). The traditional three-phase distribution power flow assigns one bus, typically the substation, as the slack bus. However, no single slack bus exists in real power systems. Also, with a significant increase in DG penetration and high resistance to reactance (R/X) ratios of dis-

tribution systems, the single slack bus assumption may distort the computed power flow results from the actual power flow. Yet, cost analysis for estimating DG impacts and subsequent planning and operating strategies are generally based on power flow simulation results (Tong et al. 2005). Thus, better loss allocation should be considered.

In this paper, a three-phase distributed slack bus model for distribution power flows is considered in order to improve power flow analysis with distributed generators and is integrated directly into DG operating cost analysis. The load distribution and the network topology play a critical role in the participation of each source with respect to servicing loads and loss. The distributed slack bus model uses network-based participation factors which quantify the amount of real power loss contributions from individual sources (Tong and Miu 2005). As such, the location of DGs is inherently captured in the power flow results and an operating cost analysis utilizes the resulting information to quantify load and loss contributions of individual sources. Subsequent simulation results show the sensitivity of operating costs with respect to DG locations.

The paper is organized as follows. First, two different slack bus models are reviewed. Then, detailed mathematical expressions for cost analysis are presented. Subsequently, based on the different slack bus models, simulation results and cost analysis are illustrated and discussed. Finally some observations and conclusions are made.

Distributed Slack Bus Models

In this section, two different slack bus models for distribution power flow are reviewed and discussed. The distributed slack bus model will be shown to be useful in reformulating DG cost analysis approaches. First, the traditional single slack bus approach is reviewed; then, the three-phase distributed slack bus is presented.

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Single Slack Bus Model

Traditional distribution power flow analysis selects the substation as the three-phase, single slack bus (Zimmerman 1995; Kersting 2002; Chen et al. 1991) which provides all the system loss. The DGs provide fixed real power outputs and are considered to make zero contributions to loss. When incorporated into power flow, DG terminal buses are generally modeled as $P|V|$ and PQ buses. While many DG models exist (Lassester et al. 2002; Zhu and Tomsovic 2002; Naka and Fukuyama 2001; Li et al. 2000), the following presents DG models used in this work.

For both types of DG buses, the total real power injection is fixed. The real power output of each phase may be different due to unbalanced network and loading conditions. Assuming the DGs are connected through an inverter (Lassester et al. 2002), the output voltage of the DG will be considered balanced. Thus, for a $P|V|$ bus model, there is only one unknown at each DG bus, the voltage phase angle e.g., θ_i^a . Therefore, only the total real power balance equation on each DG bus is required

$$P_{Gi} - \sum_{p=a}^c P_{Di}^p - \sum_{p=a}^c P_i^p = 0 \quad (1)$$

where

$$P_{Gi} = P_{Gi}^a + P_{Gi}^b + P_{Gi}^c \quad (2)$$

$$P_i^p = |V_i^p| \sum_{k=0}^n |V_k^p| [g_{ik}^p \cos(\theta_i^p - \theta_k^p) + b_{ik}^p \sin(\theta_i^p - \theta_k^p)] \quad (3)$$

where P_{Gi} =real power output of generator i ; P_{Di}^p =real power output of generator i , phase p ; and P_i^p =real load on bus i , phase p .

DGs are also subject to their reactive power limits. If a DG reaches its Q limit, it will be handled as a PQ bus. For a DG modeled as a PQ bus, the output voltage magnitude is now unknown and the total reactive power output is modeled in a similar manner as Eq. (1). Next, the distributed slack bus model will be reviewed and discussed.

Network-Based Distributed Slack Bus Model

The distributed slack bus model approach (Tong and Miu 2005) uses scalar participation factors for the substation and participating DGs whose real power outputs can be adjusted. The concept of multiphase generator domains is applied to calculate these participation factors. This results in network-based participation factors which reflect the effects of network parameters, load distributions, and generator capacities. As such, this model directly captures and quantifies the effect of the DG installation location on its output.

Only the set of participating sources including the substation and DGs with adjustable outputs are modeled using network-based participation factors. Again, assuming the DG is connected through an inverter, these participating sources are modeled as a new type of $P|V|$ buses, where the:

- Total injected real power P_{Gi} is adjustable per iteration;
- Voltage magnitudes of each phase $|V_i^p|$ are equal and specified; and
- Three voltage phase angles θ_i^p are unknown, however, the difference between any pairs are 120° ($\theta_i^a, \theta_i^b = \theta_i^a - 120^\circ, \theta_i^c = \theta_i^a + 120^\circ$).

These $P|V|$ buses provide balanced, three-phase voltage outputs, and adjustable real power inputs. Because of unbalanced loads or networks, the real power outputs for each phase, P_{Gi}^p , is not equal.

Also, the total real power output, P_{Gi} , depends on its associated load, P_{Gi}^{load} , participation factor, K_i , and the total real power system loss P_{Loss}

$$P_{Gi} = P_{Gi}^{\text{load}} + K_i P_{\text{Loss}} \quad i = 0, 1, 2, \dots, m \quad (4)$$

where

$$P_{Gi}^{\text{load}} = P_{Gi}^{\text{load},a} + P_{Gi}^{\text{load},b} + P_{Gi}^{\text{load},c} \quad (5)$$

and P_{Gi}^{load} =load associated with generator i ; $P_{Gi}^{\text{load},p}$ =load associated with generator i , phase p ; 0 =substation index; and m =number of participating DGs in the system.

From Eq. (4), we can observe that participation factors are used to quantify and distribute P_{Loss} to participating sources. Each participating source is assigned a participation factor, K_i and the sum of all participation factors is one. Therefore, participation factors are calculated as follows

$$K_i = \frac{P_{Gi}^{\text{load}}}{P_{\text{Loss}}} \quad i = 0, 1, 2, \dots, m \quad (6)$$

where

$$\sum_{i=0}^m K_i = 1 \quad (7)$$

and

$$P_{Gi}^{\text{loss}} = P_{Gi}^{\text{loss},a} + P_{Gi}^{\text{loss},b} + P_{Gi}^{\text{loss},c} \quad (8)$$

and P_{Gi}^{loss} =loss associated with generator i ; and $P_{Gi}^{\text{loss},p}$ =loss associated with generator i , phase p .

The concept of multiphase generator domain is used to determine P_{Gi}^{loss} . The concept of generator domains originates from Kirshen et al. (1997), and Strbac et al. (1998) for balanced transmission systems and it strives to distinguish loss and load associated with each participating source. In distribution systems, since the loads and network are unbalanced, the buses and branch flows supplied by the same generator may be different across phases. Thus, generator domains were extended to multiphase generator domains in Tong and Miu (2005) to associate portions of the electrical network, their loads, and losses to specific DGs

$$P_{Gi} = P_{Gi}^{\text{load}} + P_{Gi}^{\text{loss}} \quad i = 0, 1, 2, \dots, m \quad (9)$$

As such, for each phase of a participating source, associated branch loss and nodal load can be quantified and the network-based participation factor for each source can be calculated.

When integrated within a distribution power flow, the participation factors are iterative. Since the power flow with a distributed slack bus model identifies P_{Loss} as an additional unknown, an additional equation at the substation is required

$$(P_{G0}^{\text{load}} + K_0 P_{\text{Loss}}) - \sum_{p=a}^c P_{D0}^p = \sum_{p=a}^c P_0^p \quad (10)$$

where P_{D0}^p =real power load on phase p of the substation bus; and P_0^p =real power flow equation on the substation, phase p .

While the substation is still assigned as the phase reference bus, the total real power system loss is distributed through the participation factors via Eq. (4). The extended power flow equations can be solved using, for example, a Newton-Raphson solver. This distributed slack bus model of power flow can then be applied to study and re-evaluate different distribution application functions.

From the above discussions, it is observed that the real power outputs of DGs and the substation from power flow computations will be different based on different slack bus models. In the single slack bus model, DGs keep constant real power outputs and are considered to service the load throughout the power flow calculation. In the distributed slack model, the total system real power loss is iteratively distributed to participating sources based on their participation factors, which reflect their real power loss contributions corresponding to their physical location and the network load distribution. This holds when all DGs have real power output available as needed.

For cases where DGs with fixed or limited power availability exist, such as renewable energy resources, a traditional model of $P|V|$ or PQ bus modeling can be adopted for the power flow analysis. Generator domains for all DGs can be determined by post-processing the power flow voltage iterate to determine the contributions to loads and losses of each DG. Then, participation factors can be assigned to participating generators with respect to a proportionate amount of the total system loss. Thus, the different slack bus models can be applied to distinguish load and loss contributions which will impact the cost analysis of local utilities and DG operators. In the following section, the details of a cost analysis approach will be discussed.

Cost Analysis

Cost analysis is the driving method for utilities and DG owners to estimate their profits, to assist in determining DG installation and operation strategies, and to improve their competitive capabilities. In this section, operating cost analysis based on the economic profit formula (McConnell and Bruce 2002) is investigated with detailed mathematical expressions of revenue and cost calculations provided. These expressions directly incorporate the distributed slack bus model parameters for DGs and the substation. As such, the dollar cost impacts of different slack bus models can be quantified. Since distinguishing load and loss contributions of each source is very important to correctly account for revenue and cost, the distributed slack bus model provides advantages over the single slack bus model for cost analysis.

For each source i , the economic profit (EP_{*i*}) is the difference between its total revenue (TR_{*i*}) and its total cost (TC_{*i*})

$$EP_i = TR_i - TC_i \quad (11)$$

Proposed methods to account for revenue and cost are varied in electric power markets (Willis and Scott 2000; PA PUC 2005; Commerce Energy 2006); and methods proposed for distribution systems are also under development. In this paper, the goal of this section is to demonstrate the possibility for different cost analysis approaches when a distributed slack bus model that is capable of distinguishing load and loss contributions is used.

Utilities and DGs supply electricity to customers; their revenues are primarily based on customers' energy consumption. Therefore, the total revenue of a source, TR_{*i*}, can be expressed as follows

$$TR_i = \sum_{l=1}^{n_l} \sum_{j \in D_i^l} B_j^l P_j^l T_j^l \quad (12)$$

where n_l =number of load levels; D_i^l =set of loads served by source i at load level l ; B_j^l =price of real power load j at load level l (\$/kWh); P_j^l =real power delivered to load j at load level l (kW); and T_j^l =duration of time of load j in hours at load level l (h). Here

P_j^l can be assigned using the models from Eqs. (4) and (9) through a power flow at load level l and

$$\sum_{j \in D_i^l} P_j^l = P_{Gi}^{\text{load},l} \quad (13)$$

with $P_{Gi}^{\text{load},l}$ representing the load associated with generator i at load level l .

Closer evaluation shows that Eq. (12) holds for the substation. However, using Eq. (12) results in the DG absorbing the entire cost for producing its associated loss contributions. Since the network-based distributed slack bus model can distinguish these loss contributions and these contributions enhance the distribution network capability and often improve distribution network efficiency, loss contributions from DGs should be encouraged. As such, network operators could also represent revenue sources to DGs by providing financial compensation for the losses provided. Thus, for DGs, Eq. (12) is modified as

$$TR_i = \sum_{l=1}^{n_l} \sum_{j \in D_i^l} B_j^l P_j^l T_j^l + \sum_{l=1}^{n_l} A_u^l P_{Gi}^{\text{loss},l} T_{Gi}^l \quad (14)$$

where A_u^l =price utilities' payment to DG real power loss contribution at load level l (\$/kWh); $P_{Gi}^{\text{loss},l}$ =loss associated with generator i at load level l (kWh); and T_{Gi}^l =duration of the duration of operating time of DG k at load level l (h).

The payment of loss contributions to DGs is accounted for as a part of total cost of utilities. Thus the total cost of the local utilities has three parts: payments to generation and transmission systems including energy charges and capacity charges and payment to DG operators for loss contributions. The energy charge for a distribution utility C_{en} is a function of the price per kilowatt-hour (kWh) and the kWh of real power injection from the transmission system at the substation

$$C_{\text{en}} = \sum_{l=1}^{n_l} A_e^l T_l P_{\text{Sinj}}^l = \sum_{l=1}^{n_l} A_e^l \left[T_l (P_{\text{Load}}^l + P_{\text{Loss}}^l) - \sum_{k=1}^m T_{Gk}^l P_{Gk}^l \right] \quad (15)$$

where A_e^l =real power price on the substation at load level l (\$/kW·h); T_l =duration of time of load level l (h); P_{Sinj}^l =real power injection on the substation from the transmission system at load level l (kW); P_{Load}^l =total system real power load at load level l (kW); P_{Loss}^l =total system real power loss at load level l (kW); T_{Gk}^l =duration of operating time of DG k at load level l (h); and P_{Gk}^l =real power injection from DG k at load level l (kW).

The capacity charge, C_{ca} , is charged at the maximum apparent power on the substation during a period of time

$$C_{\text{ca}} = [A_c J (365 * 24)] S_{\text{inj}}^{\text{max}} \sum_{l=1}^{n_l} T_l \quad (16)$$

where A_c =price of capacity charge on the substation (\$/kVA/year); and $S_{\text{inj}}^{\text{max}}$ =maximum apparent power injection on the substation during whole period of time (kVA).

Payments to DG operators for loss contributions, C_{lo} , are the sum of payments to participating DGs

$$C_{\text{lo}} = \sum_{l=1}^{n_l} \sum_{i=1}^m A_u^l P_{Gi}^{\text{loss},l} T_{Gi}^l \quad (17)$$

This payment effectively quantifies and encourages DG placements in locations where the network experiences higher losses.

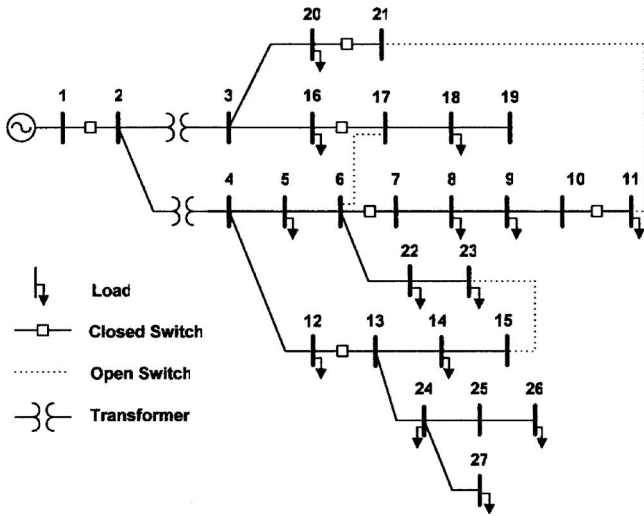


Fig. 1. One-line diagram of three-phase distribution system

However, it also results in reducing payments of the distribution utility to the transmission and generation operators. Thus, from the distribution system standpoint, ideally this payment rate should not be higher than the nodal price on the substation. In summary, the total cost to a distribution utility is the sum of its energy charge, capacity charge and payment to DGs' loss contributions

$$TC_{sub} = C_{en} + C_{ca} + C_{lo} \quad (18)$$

For DGs, their operating cost mainly comes from their power generating cost. Thus, the total cost of a DG, without connection tariff, can be expressed as

$$TC_{Gk} = \sum_{l=1}^{n_l} A_{Gk}^l P_{Gk}^l T_{Gk}^l \quad (19)$$

where A_{Gk}^l =average real power generating price of DG k at load level l (\$/kWh); and P_{Gk}^l =real power injection from DG k at load level l (kWh).

The cost of generation will increase as the generator real power output moves away from its optimal efficiency operating point. To capture this, the average real power price A_{Gk}^l for generating P_{Gk}^l can be determined from a lookup table or functional expression if available.

From the above revenue and cost expressions, we can observe that the revenue of each source is relative to the associated load it services. In addition, with a distributed slack bus model, cost analysis can be fine tuned to identify different costs/revenues for load contributions and loss contributions. Thus, the distributed slack bus model allows more detailed profit and cost evaluations of individual sources. If a single slack model is used where all the system loss is assigned to the substation, this may unfairly punish the network operators. However, using computational tools that can distinguish loads and losses, fairer pricing can be achieved with DGs receiving compensation for network benefits they provide. Assuming the payment rate toward loss contributions is less than the transmission system charge; distribution utility profits can actually increase with appropriate compensation to DG operators.

In addition, the network-based approach to assigning slack and associated cost analysis can better capture and identify optimal locations to install distributed generators. These models and cost

Table 1. Power Flow Results for 26-Bus Test System without DGs

| Parameters | Test results |
|--|--------------|
| Power flow results | |
| Substation real power output (kW) | 6,939.49 |
| Substation apparent power (kVA) | 7,453.70 |
| Total system real power load (kW) | 6,659.38 |
| Real power load of bus 3 downstream (kW) | 3,080.24 |
| Real power load of bus 4 downstream (kW) | 3,579.14 |
| Total system real power loss (kW) | 280.11 |
| Real power loss of bus 3 downstream (kW) | 13.53 |
| Real power loss of bus 4 downstream (kW) | 255.96 |
| Cost analysis results | |
| Total annual revenue (United States Dollars, \$) | 4,958,574 |
| Energy charge (\$) | 4,559,247 |
| Capacity charge (\$) | 335,420 |
| Payment to DG (\$) | 0 |
| Total annual economic profit (\$) | 63,907 |
| Cost to supply loss (\$) | 184,034 |

analysis approaches have been implemented in MATLAB; and, in the next section, detailed simulation results will be presented to show the impacts of cost analysis and placement strategies using different slack bus models.

Numerical Results

In this section, a MATLAB based unbalanced-phase power flow solver and cost analysis program were implemented and utilized to perform cost analysis using different slack bus models. To clearly show the impacts from the distributed slack bus models for DGs, one system load level is applied for a 1-year time period. It is assumed that the DG is owned and operated by an entity independent of the distribution utility. Cost evaluations for DG installations at different locations will be studied; in addition, various levels of DG output will be investigated. For each case, the following cost parameters are used for this section's analysis (Willis and Scott 2000; PA PUC 2005); it is noted that, other cost schemes can be readily incorporated in the program:

1. Flat energy charge for all customers within distribution system: 0.085 \$/kWh;
2. Electricity charge on the substation from transmission for the local utility, including two parts:
 - Electricity energy charge: 0.075 \$/kWh;
 - Electricity capacity charge: 45 \$/kVA/year; and
3. The average electricity generating cost of the DG: 0.07 \$/kWh.

A 26-bus test distribution system will be used and its one-line diagram is shown in Fig. 1. The network was designed from a portion of an existing system with real network parameters. For the simulations, all loads are treated as constant PQ loads and the total system load is 6.659 MW and 2.539 MVAR. Using a single slack bus, unbalanced distribution power flow solver, the total system real power loss, P_{Loss} is 280 kW or 4.2% of real power injection at the substation. The transformer between Bus 2 and Bus 3 services 3.08 MW and 0.95 MVAR high density loads. The transformer between Bus 2 and Bus 4 services 3.58 MW and 1.58 MVAR dispersed loads in a commercial and residential area. Without DG installed, 255.96 kW (91.4% of total system real power loss) occurs in the commercial and residential area, and

Table 2. Power Flow Results for 26-Bus Test System without DGs: per Unit ($|V|$ p.u.) Voltage Magnitude and Voltage Angle (Θ in Degrees) for Phases a, b, c

| Bus number | $ Va $ p.u. | Θ_a (degrees) | $ Vb $ p.u. | Θ_b (degrees) | $ Vc $ p.u. | Θ_c (degrees) |
|------------|-------------|-------------------------|-------------|-------------------------|-------------|-------------------------|
| 1 | 1 | 0 | 1 | -120 | 1 | 120 |
| 2 | 0.999873125 | -0.004307348 | 0.999875168 | -120.0042584 | 0.999874515 | 119.9957622 |
| 3 | 0.993373158 | -0.969781444 | 0.993409923 | -120.9495600 | 0.993399944 | 119.0485849 |
| 4 | 0.989360785 | -1.179471596 | 0.989454644 | -121.1802100 | 0.989519308 | 118.8156858 |
| 5 | 0.973569424 | -0.825192692 | 0.973705067 | -120.8287272 | 0.973736187 | 119.1644193 |
| 6 | 0.957192130 | -0.503171977 | 0.957285106 | -120.5077971 | 0.957439750 | 119.4802509 |
| 7 | 0.957192130 | -0.503171977 | 0.957285106 | -120.5077971 | 0.957439750 | 119.4802509 |
| 8 | 0.923613056 | 0.006381575 | 0.923575934 | -120.0015313 | 0.924093177 | 119.9702951 |
| 9 | 0.907869606 | 0.286417473 | 0.907905120 | -119.7277591 | 0.908326190 | 120.2373119 |
| 10 | 0.902447039 | 0.418948395 | 0.902483226 | -119.5952313 | 0.902906509 | 120.3697374 |
| 11 | 0.902447039 | 0.418948395 | 0.902483226 | -119.5952313 | 0.902906509 | 120.3697374 |
| 12 | 0.945801296 | -0.324547201 | 0.945722954 | -120.3436052 | 0.945670028 | 119.6416677 |
| 13 | 0.945801296 | -0.324547201 | 0.945722954 | -120.3436052 | 0.945670028 | 119.6416677 |
| 14 | 0.932689876 | -0.010168897 | 0.932610518 | -120.0291304 | 0.932556151 | 119.9561621 |
| 15 | 0.932689876 | -0.010168897 | 0.932610518 | -120.0291304 | 0.932556151 | 119.9561621 |
| 16 | 0.990085203 | -1.113288616 | 0.990386812 | -121.0814718 | 0.990220664 | 118.9221123 |
| 17 | 0.990085203 | -1.113288616 | 0.990386812 | -121.0814718 | 0.990220664 | 118.9221123 |
| 18 | 0.988430283 | -1.135080650 | 0.988732604 | -121.1032803 | 0.988566522 | 118.9003216 |
| 19 | 0.988430283 | -1.135080650 | 0.988732604 | -121.1032803 | 0.988566522 | 118.9003216 |
| 20 | 0.987414031 | -1.053598070 | 0.987451321 | -121.0334503 | 0.987442339 | 118.9647481 |
| 21 | 0.987414031 | -1.053598070 | 0.987451321 | -121.0334503 | 0.987442339 | 118.9647481 |
| 22 | 0.945098928 | -0.111123842 | 0.945193549 | -120.1157730 | 0.945349902 | 119.8721679 |
| 23 | 0.936647575 | 0.167543214 | 0.936743417 | -119.8371611 | 0.936901000 | 120.1507048 |
| 24 | 0.915503648 | 0.159537050 | 0.915174188 | -119.8873666 | 0.914952842 | 120.0818682 |
| 25 | 0.907781609 | 0.306360180 | 0.907441412 | -119.7400838 | 0.907128256 | 120.2283152 |
| 26 | 0.900969946 | 0.438060286 | 0.900620301 | -119.6079677 | 0.900226040 | 120.3597074 |
| 27 | 0.904791645 | 0.315985445 | 0.904373899 | -119.7615426 | 0.904288683 | 120.2284207 |

only 13.53 kW (4.8% of the total loss) occurs in the high density load area due to shorter branches and lower network resistances. The local utility acquires electricity from the transmission system to supply loads and loss within the system. Using the cost parameters above results in the distribution utility incurring an annual revenue of approximately \$4,958,000 United States Dollars, annual cost \$4,894,700, and annual profit \$63,900. A summary of the power flow results and annual cost analysis for the system without DG is shown in Tables 1 and 2.

In the following examples, three cases will be investigated where penetration is defined as the percentage of the target real power output to the total system real power load:

Case 1. The DG is installed at Bus 3 with different DG penetration;

Case 2. The DG is installed at Bus 4 with different DG penetration; and

Case 3. The DG is installed at different locations with the same DG penetration.

It is noted that the above locations for the DG placements were arbitrarily selected and that the modeling and cost analysis methodology applies for any placement location. It is expected that the ability of the distributed slack bus model to quantify loss and loads will yield significant differences in cost analysis compared to a traditional single slack bus power flow. Thus, these tools can

Table 3. Power Flow Results with DG on Bus 3

| Parameters | Single slack bus model | | | Distributed slack bus model | | |
|--------------------------------------|------------------------|----------|----------|-----------------------------|----------|----------|
| | 500 | 1,000 | 1,500 | 500 | 1,000 | 1,500 |
| real power load serviced by DG (kW) | | | | | | |
| DG real power output (kW) | 500 | 1,000 | 1,500 | 502.17 | 1,004.33 | 1,506.50 |
| Substation real power output (kW) | 6,437.80 | 5,936.89 | 5,436.17 | 6,435.63 | 5,932.55 | 5,429.66 |
| Substation apparent power (kVA) | 6,618.93 | 6,139.62 | 5,664.73 | 6,616.84 | 6,135.48 | 5,658.59 |
| Total system real power loss (kW) | 278.42 | 277.51 | 276.79 | 278.41 | 277.51 | 276.78 |
| Real power loss serviced by DG (kW) | 0 | 0 | 0 | 2.17 | 4.33 | 6.50 |
| Real power loss serviced by sub (kW) | 278.42 | 277.51 | 276.79 | 276.25 | 273.17 | 270.28 |
| Participation factor for DG | 0 | 0 | 0 | 0.0078 | 0.0156 | 0.0235 |
| Participation factor for substation | 1 | 1 | 1 | 0.9922 | 0.9844 | 0.9765 |

Table 4. Cost Analysis for Local Utility with DG on Bus 3 (Units: US\$)

| Parameters | Single slack bus model | | | Distributed slack bus model | | |
|--|------------------------|-----------|-----------|-----------------------------|-----------|-----------|
| | 500 | 1,000 | 1,500 | 500 | 1,000 | 1,500 |
| real power load serviced by DG (kW) | | | | | | |
| Total annual revenue | 4,586,274 | 4,213,974 | 3,841,674 | 4,586,274 | 4,213,974 | 3,841,674 |
| Energy charge | 4,229,634 | 3,900,538 | 3,571,563 | 4,228,207 | 3,897,686 | 3,567,287 |
| Capacity charge | 297,852 | 276,283 | 254,913 | 297,758 | 276,097 | 254,636 |
| Payment to DG with Au=0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Payment to DG with Au=0.03 | 0 | 0 | 0 | 570 | 1,139 | 1,708 |
| Payment to DG with Au=0.075 | 0 | 0 | 0 | 1,424 | 2,848 | 4,271 |
| Total annual economic profit with Au=0 | 58,788 | 37,153 | 15,198 | 60,309 | 40,191 | 19,751 |
| Total annual economic profit with Au=0.03 | 58,788 | 37,153 | 15,198 | 59,740 | 39,053 | 18,043 |
| Total annual economic profit with Au=0.075 | 58,788 | 37,153 | 15,198 | 58,885 | 37,344 | 15,480 |
| Cost to supply loss with Au=0 | 182,921 | 182,326 | 181,851 | 181,495 | 179,474 | 177,574 |
| Cost to supply loss with Au=0.03 | 182,921 | 182,326 | 181,851 | 182,064 | 180,613 | 179,283 |
| Cost to supply loss with Au=0.075 | 182,921 | 182,326 | 181,851 | 182,919 | 182,321 | 181,845 |

be utilized for rigorous optimization programs which determine optimal DG placement through minimal cost evaluations. In the results that follow, the impact of DG locations will be illustrated; and it is expected that, for this case, installations in areas of the distribution network with more dispersed loads will illustrate larger differences in cost than installations within high density load areas.

Case 1: DG Installed at Bus 3

One DG is sited at Bus 3 to service 0.5, 1, and 1.5 MW real power load, respectively. Bus 3 is located near the high density loads. Power flow results obtained from a single slack bus model and distributed slack bus model are displayed in Table 3. The real power system loss is modestly reduced when the DG penetration is increased, with the 1,500 kW set point representing 20% penetration. Using the traditional power flow approach of a single slack bus model, the DG is treated to have no contribution to system real power loss. While using the distributed slack bus model, the DG's participation factor, K , and associated real power loss contribution is displayed.

For the cost analysis based on Eq. (17), three different rates are applied for the utility's payment to DG operators for loss contributions $A_u = \$0, \$0.03, \text{ and } \$0.075$, respectively. Table 4 displays the resulting cost analysis from the system/distribution utility standpoint. Fig. 2 displays the annual economic profit of the distribution utility for the various amounts of DG penetration and A_u . It is observed that:

1. For both slack bus models, the total annual revenue, consumption charge, capacity charge, and economic profit of the utility decrease with increased amounts of DG penetration; and
2. Using the distributed slack bus model and for each A_u , the utility has higher profit than using the single slack bus model because the DG supplies a portion of the losses at a cost to the utility less than if the utility purchased the same amount from the substation. The delineation of the amount of loss allows for this accounting to be performed. Thus, in Fig. 2, the profit curves with \$0.03 and \$0.075 rates are located between the curve of the distributed slack bus model with zero rate and the curve of the single slack bus model.

In Table 5, cost analysis results with respect to the DG are displayed. The total annual revenue, annual cost, and economic profit of the DG increase with the increase in DG penetration for

both slack bus models. The loss contribution payments from utilities further increased the DG's profit using the distributed slack bus model.

Case 2: DG Installed at Bus 4

One DG is sited at Bus 4 to service 0.5, 1, and 1.5 MW real power load, respectively. Bus 4 is located closer to the dispersed loads. The simulation results using different slack bus models are shown in Table 6. As in Case 1, the real power system loss of this case is also slightly reduced with increasing DG penetration for both slack bus models, and the value of the DG's participation factor is increasing with DG penetration. However, the participation factor assigned to the DG on Bus 4 is much larger than that of Bus 3 at the same penetration level. The larger factors reflect the higher percentage of system real power loss occurring downstream of Bus 4 which is identified as the generator's domain. Thus, the DG installed on Bus 4 is identified to make larger real power loss contributions than a DG placed at Bus 3.

As a result, the cost analysis from the distribution utility standpoint is quite interesting and displayed in Table 7. For both slack bus models, the total annual revenue, energy charge, and capacity

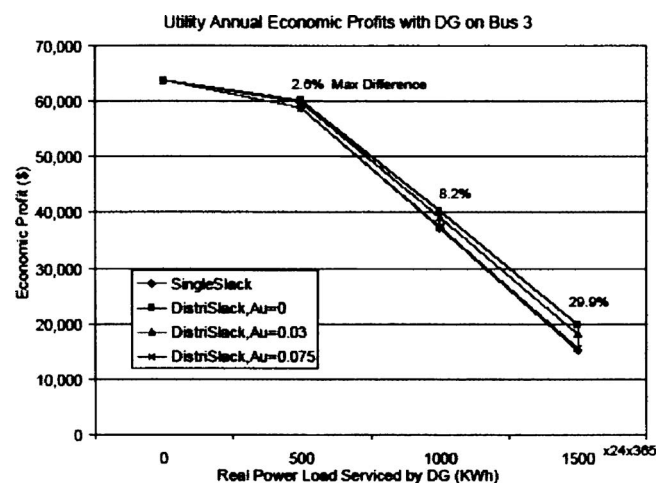


Fig. 2. Annual economic profit for local utility with distributed generator (DG) at Bus 3: with maximum difference between lines of single slack bus model and distributed slack bus model with $A_u = 0$

Table 5. Cost Analysis for Distributed Generator on Bus 3 (Units: US\$)

| Parameters | Single slack bus model | | | Distributed slack bus model | | |
|---|------------------------|---------|-----------|-----------------------------|-----------|-----------|
| | 500 | 1000 | 1,500 | 500 | 1,000 | 1,500 |
| real power load serviced by DG (kW) | | | | | | |
| Total annual revenue with $A_u=0$ | 372,300 | 744,600 | 1,116,900 | 372,300 | 744,600 | 1,116,900 |
| Total annual revenue with $A_u=0.03$ | 372,300 | 744,600 | 1,116,900 | 372,869.6 | 745,739 | 1,118,608 |
| Total annual revenue with $A_u=0.075$ | 372,300 | 744,600 | 1,116,900 | 373,723.9 | 747,447.5 | 1,121,171 |
| Total annual cost | 306,600 | 613,200 | 919,800 | 307,929 | 615,858 | 923,786 |
| Total annual economic profit with $A_u=0$ | 65,700 | 131,400 | 197,100 | 64,371 | 128,742 | 193,114 |
| Total annual economic profit with $A_u=0.03$ | 65,700 | 131,400 | 197,100 | 64,941 | 129,881 | 194,822 |
| Total annual economic profit with $A_u=0.075$ | 65,700 | 131,400 | 197,100 | 65,795 | 131,590 | 197,384 |
| Cost to supply loss with $A_u=0$ | 0 | 0 | 0 | 1,329 | 2,658 | 3,986 |
| Cost to supply loss with $A_u=0.03$ | 0 | 0 | 0 | 759 | 1,519 | 2,278 |
| Cost to supply loss with $A_u=0.075$ | 0 | 0 | 0 | -95 | -190 | -285 |

charge of the utility decrease with increasing DG penetration. As we expected, regardless of the model, the DG installed on Bus 4 has more beneficial impacts on the cost analysis than the same DG on Bus 3. However, different impacts on the total annual economic profits of the utility are observed with increases in DG penetration (see Fig. 3):

1. Using the single slack bus model, the total annual economic profit of the utility first slightly increases with DG penetration and then decreases with continued increase in DG penetration;
2. Using the distributed slack bus model, a similar increase in profit with certain levels of DG penetration is also observed. These initial increases in profit imply that the distribution

utility can still economically benefit from proper DG placement and sizing through loss reduction and reduced capacity charges even though they may no longer supply some portions of their original customers; and

3. Using the distributed slack bus model with, $A_u=0$ zero payments to DG loss contributions, the total annual economic profit of the utility consistently increases with DG penetration.

These different results in profits may cause different behaviors: for the same location, the utility may encourage higher DG penetration based on the analysis using the distributed slack bus model than using the single slack bus model.

In Table 8, the total annual revenue, annual cost and economic

Table 6. Power Flow Results with DG on Bus 4

| Parameters | Single slack bus model | | | Distributed slack bus model | | |
|--------------------------------------|------------------------|----------|----------|-----------------------------|----------|----------|
| | 500 | 1,000 | 1,500 | 500 | 1,000 | 1,500 |
| real power load serviced by DG (kW) | | | | | | |
| DG real power output (kW) | 500 | 1,000 | 1,500 | 534.89 | 1,069.78 | 1,604.67 |
| Substation real power output (kW) | 6,430.83 | 5,929.67 | 5,428.69 | 6,395.85 | 5,859.73 | 5,323.83 |
| Substation apparent power (kVA) | 6,482.17 | 5,988.73 | 5,497.20 | 6,447.68 | 5,920.00 | 5,394.60 |
| Total system real power loss (kW) | 271.45 | 270.29 | 269.31 | 271.36 | 270.14 | 269.12 |
| Real power loss supplied by DG (kW) | 0 | 0 | 0 | 34.89 | 69.78 | 104.67 |
| Real power loss supplied by sub (kW) | 271.45 | 270.29 | 269.31 | 236.47 | 200.36 | 164.45 |
| Participation factor for DG | 0 | 0 | 0 | 0.1286 | 0.2583 | 0.3889 |
| Participation factor for substation | 1 | 1 | 1 | 0.8714 | 0.7417 | 0.6111 |

Table 7. Cost Analysis for Local Utility with DG on Bus 4 (Units: \$)

| Parameters | Single slack bus model | | | Distributed slack bus model | | |
|---|------------------------|-----------|-----------|-----------------------------|-----------|-----------|
| | 500 | 1,000 | 1,500 | 500 | 1,000 | 1,500 |
| real power load serviced by DG (kW) | | | | | | |
| Total annual revenue | 4,586,274 | 4,213,974 | 3,841,674 | 4,586,274 | 4,213,974 | 3,841,674 |
| Energy charge | 4,225,057 | 3,895,792 | 3,566,647 | 4,202,074 | 3,849,846 | 3,497,756 |
| Capacity charge | 291,697 | 269,493 | 247,370 | 290,146 | 266,400 | 242,760 |
| Payment to DG with $A_u=0$ | 0 | 0 | 0 | 0 | 0 | 0 |
| Payment to DG with $A_u=0.03$ | 0 | 0 | 0 | 9,170 | 18,339 | 27,508 |
| Payment to DG with $A_u=0.075$ | 0 | 0 | 0 | 22,925 | 45,848 | 68,771 |
| Total annual economic profit with $A_u=0$ | 69,520 | 48,689 | 27,657 | 94,054 | 97,728 | 101,158 |
| Total annual economic profit with $A_u=0.03$ | 69,520 | 48,689 | 27,657 | 84,885 | 79,389 | 73,649 |
| Total annual economic profit with $A_u=0.075$ | 69,520 | 48,689 | 27,657 | 71,130 | 51,880 | 32,387 |
| Cost to supply loss with $A_u=0$ | 178,344 | 177,579 | 176,935 | 155,361 | 131,634 | 108,044 |
| Cost to supply loss with $A_u=0.03$ | 178,344 | 177,579 | 176,935 | 164,532 | 149,973 | 135,552 |
| Cost to supply loss with $A_u=0.075$ | 178,344 | 177,579 | 176,935 | 178,287 | 177,482 | 176,815 |

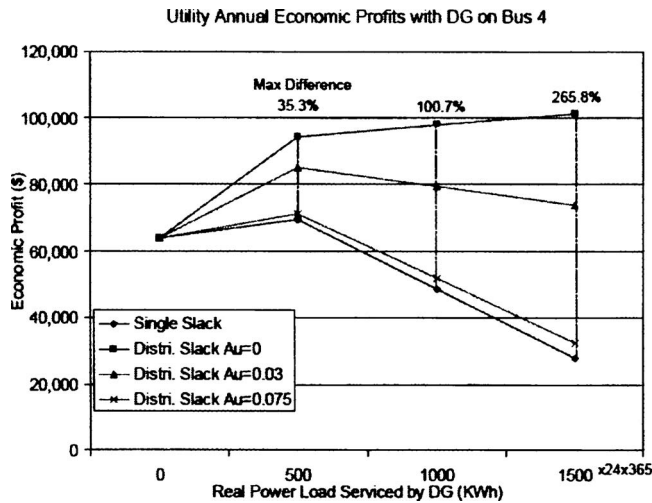


Fig. 3. Annual economic profit for local utility with DG at Bus 4: with maximum difference between lines of single slack bus model and distributed slack bus model with $A_u=0$

profit of the DG also increase with an increase in DG penetration for both models. For the single slack bus model, Case 1 and Case 2 yield the same cost analysis. As expected, the annual profit results for the distributed slack bus model are significantly different from those in Case 1. While the total system loss decreases with increases in DG penetration, the DG contribution to total system loss increases with a distributed slack bus model. As a consequence, the rate at which a DG may be compensated for providing loss contributions is significant; and its impact on profit can also be seen in Fig. 3. Therefore, some DG installations may be specifically identified for improved electrical and cost efficiency.

Case 3: DG at Different Locations

To study the effects of different DG placement locations using the different slack bus models, a DG is selected to service 1.5 MW load on Bus 6, Bus 13, and Bus 19, respectively. From Fig. 4 and Table 9, it can be observed that the DG's loss contribution, reflected by its participation factors, is significantly impacted by its location. To service the same amount of load, the DG participation factors are significantly different: 0.3163 at Bus 6 versus 0.027 at Bus 19. When the DG is installed at Bus 6, the system real power loss is reduced from 280.11 to 237.55 kW with a

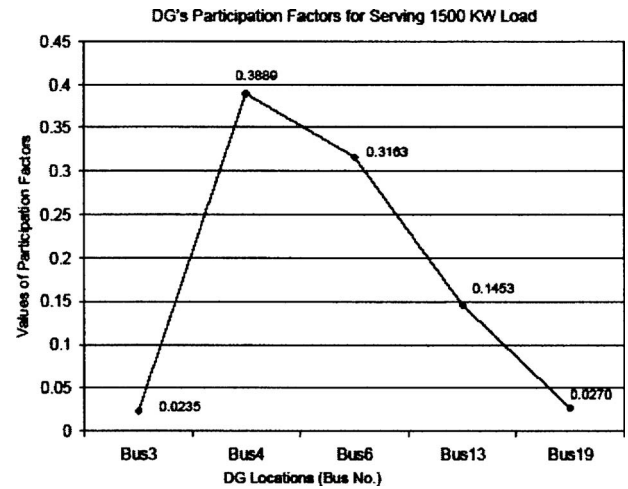


Fig. 4. Participation factors of DG's Serving 1,500 kW load from different locations

single slack bus model and to 227.23 kW with the distributed slack bus model. When the DG is at Bus 19, its participation factor is 0.027 and the system real power loss is reduced however by a much smaller level.

Tables 10 and 11 present the cost analysis from the utility viewpoint and from the DG standpoint, respectively. A plot for the utility profits is displayed in Fig. 5. Results illustrate significant differences between costs when losses are not distributed and one where a distributed slack bus model is employed. From Tables 10 and 11, it is possible to quantify cost differences between locations; therefore flat rate interconnection charges may not be optimal to either the distribution utility or the DG operator. Thus network-based participation factors used in the distributed slack bus model can quantify loss contributions of participating sources and provides an advantage for improved cost analysis.

From the above simulations, the following summary of comments and observations follow:

1. The network-based participation factors of the distributed slack bus model reflect the ratios of participating sources' real power loss contributions to the system real power loss;
2. Consequently, the approach is able to quantify the different impacts on system loss based on different DG locations, with larger participation factors indicating a larger impact on system loss;
3. Local loss contributions and the release of system capacity

Table 8. Cost Analysis for DG on Bus 4 (Units: \$)

| Parameters | Single slack bus model | | | Distributed slack bus model | | |
|---|------------------------|---------|-----------|-----------------------------|----------|-----------|
| | 500 | 1,000 | 1,500 | 500 | 1,000 | 1,500 |
| real power load serviced by DG (kW) | | | | | | |
| Total annual revenue with $A_u=0$ | 372,300 | 744,600 | 1,116,900 | 372,300 | 744,600 | 1,116,900 |
| Total annual revenue with $A_u=0.03$ | 372,300 | 744,600 | 1,116,900 | 381470.1 | 762939.3 | 1144408 |
| Total annual revenue with $A_u=0.075$ | 372,300 | 744,600 | 1,116,900 | 395225.2 | 790448.2 | 1185671 |
| Total annual cost | 306,600 | 613,200 | 919,800 | 327,997 | 655,992 | 983,986 |
| Total annual economic profit with $A_u=0$ | 65,700 | 131,400 | 197,100 | 44,303 | 88,608 | 132,914 |
| Total annual economic profit with $A_u=0.03$ | 65,700 | 131,400 | 197,100 | 53,473 | 106947.6 | 160422.1 |
| Total annual economic profit with $A_u=0.075$ | 65,700 | 131,400 | 197,100 | 67,228 | 134456.5 | 201684.7 |
| Cost to supply loss with $A_u=0$ | 0 | 0 | 0 | 21,397 | 42,792 | 64,186 |
| Cost to supply loss with $A_u=0.03$ | 0 | 0 | 0 | 12,227 | 24,452 | 36,678 |
| Cost to supply loss with $A_u=0.075$ | 0 | 0 | 0 | -1,528 | -3,057 | -4,585 |

Table 9. Power Flow Results for DG at Different Locations to Service 1.5 MW Load

| Parameters DG location | Single slack bus model | | | Distributed slack bus model | | |
|--------------------------------------|------------------------|----------|----------|-----------------------------|----------|----------|
| | Bus 6 | Bus 13 | Bus 19 | Bus 6 | Bus 13 | Bus 19 |
| DG real power output (kW) | 1,500 | 1,500 | 1,500 | 1,571.87 | 1,533.83 | 1,507.55 |
| Substation real power output (kW) | 5,396.93 | 5,402.76 | 5,439.32 | 5,314.75 | 5,358.41 | 5,431.85 |
| Substation apparent power (kVA) | 5,422.89 | 5,412.62 | 6,002.41 | 1,571.87 | 5,378.27 | 5,997.63 |
| Total system real power loss (kW) | 237.55 | 243.38 | 279.94 | 227.23 | 232.85 | 280.03 |
| Real power loss supplied by DG (kW) | 0 | 0 | 0 | 71.87 | 33.83 | 7.55 |
| Real power loss supplied by sub (kW) | 237.55 | 243.38 | 279.94 | 155.37 | 199.03 | 272.47 |
| Participation factor for DG | 0 | 0 | 0 | 0.3163 | 0.1453 | 0.0270 |
| Participation factor for substation | 1 | 1 | 1 | 0.6837 | 0.8547 | 0.9730 |

- from DGs can improve utilities' profits. Reasonable payment rates to DG operators for loss contributions can bring benefits for both utilities and DG owners;
4. For the local utility:
 - a. The cost analysis results are significantly different using different slack bus models;
 - b. Penetration levels of DGs will affect the economic profit of the local utility. Increasing and decreasing the profit of the local utility are both possible by increasing DG penetration;
 - c. Locations of DG installation also impact the profit of the local utility; and
 5. For the DG:
 - a. The cost analysis results are significantly different using slack bus model;
 - b. Penetration of DG will affect the profit of the DG; and
 - c. The selected location for the DG greatly affects the profit of the DG when a distributed slack bus model is used; while the profit is lower than if the losses are not distributed this method for determining nonload service benefits to the system would be useful to properly determine interconnection charges.
- Through these studies, it can be extrapolated that a distributed slack bus approach in power flow analysis and cost analysis can

Table 10. Cost Analysis for Local Utility with DG at Different Locations to Service 1.5 MW Load (Unit: \$)

| Parameters DG location | Single slack bus model | | | Distributed slack bus model | | |
|--|------------------------|-----------|-----------|-----------------------------|-----------|-----------|
| | Bus 6 | Bus 13 | Bus 19 | Bus 6 | Bus 13 | Bus 19 |
| Total annual revenue | 3,841,674 | 3,841,674 | 3,841,674 | 3,841,674 | 3,841,674 | 3,841,674 |
| Energy charge | 3,545,782 | 3,549,615 | 3,573,635 | 3,491,789 | 3,520,474 | 3,568,727 |
| Capacity charge | 244,030 | 243,568 | 270,109 | 241,495 | 242,022 | 269,893 |
| Payment to DG with Au=0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Payment to DG with Au=0.03 | 0 | 0 | 0 | 18,887 | 8,890 | 1,985 |
| Payment to DG with Au=0.075 | 0 | 0 | 0 | 47,217 | 22,224 | 4,962 |
| Total annual economic profit with Au=0 | 51,862 | 48,491 | -2,070 | 108,390 | 79,178 | 3,054 |
| Total annual economic profit with Au=0.03 | 51,862 | 48,491 | -2,070 | 89,504 | 70,289 | 1,069 |
| Total annual economic profit with Au=0.075 | 51,862 | 48,491 | -2,070 | 61,174 | 56,954 | -1,908 |
| Cost to supply loss with Au=0 | 156,070 | 159,903 | 183,923 | 102,076 | 130,761 | 179,015 |
| Cost to supply loss with Au=0.03 | 156,070 | 159,903 | 183,923 | 120,963 | 139,651 | 180,999 |
| Cost to supply loss with Au=0.075 | 156,070 | 159,903 | 183,923 | 149,293 | 152,985 | 183,977 |

Table 11. Cost Analysis for DG to Service 1.5 MW Load from Different Locations (Units \$)

| Parameters DG location | Single slack bus model | | | Distributed slack bus model | | |
|--|------------------------|-----------|-----------|-----------------------------|-----------|-----------|
| | Bus 6 | Bus 13 | Bus 19 | Bus 6 | Bus 13 | Bus 19 |
| Total annual revenue with Au=0 | 1,116,900 | 1,116,900 | 1,116,900 | 1,116,900 | 1,116,900 | 1,116,900 |
| Total annual revenue with Au=0.03 | 1,116,900 | 1,116,900 | 1,116,900 | 1,135,787 | 1,125,790 | 1,118,885 |
| Total annual revenue with Au=0.075 | 1,116,900 | 1,116,900 | 1,116,900 | 1,164,117 | 1,139,124 | 1,121,862 |
| Total annual cost | 919,800 | 919,800 | 919,800 | 963,869 | 940,542 | 924,431 |
| Total annual economic profit with Au=0 | 197,100 | 197,100 | 197,100 | 153,031 | 176,358 | 192,469 |
| Total annual economic profit with Au=0.03 | 197,100 | 197,100 | 197,100 | 171,918 | 185,247 | 194,454 |
| Total annual economic profit with Au=0.075 | 197,100 | 197,100 | 197,100 | 200,248 | 198,582 | 197,431 |
| Cost to supply loss with Au=0 | 0 | 0 | 0 | 44,069 | 20,742 | 4,631 |
| Cost to supply loss with Au=0.03 | 0 | 0 | 0 | 25,182 | 11,853 | 2,646 |
| Cost to supply loss with Au=0.075 | 0 | 0 | 0 | -3,148 | -1,482 | -331 |

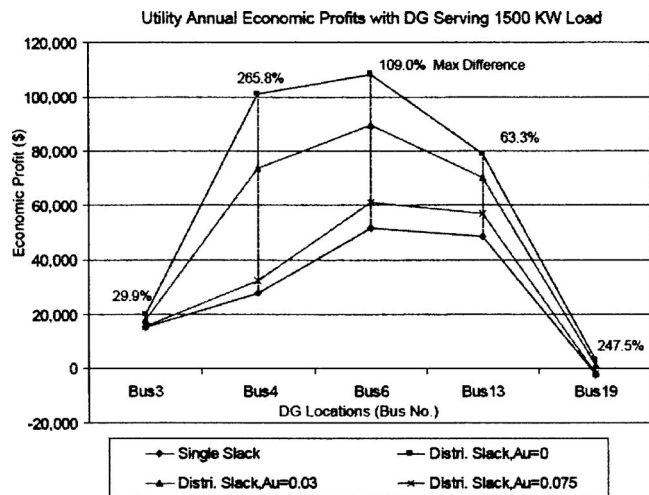


Fig. 5. Annual economic profit for local utility with distributed generators at different locations serving 1,500 kW load: with maximum difference between lines of single slack bus model and distributed slack bus model with $A_u=0$

significantly impact distribution application functions such as distribution generator placement and control problems.

Conclusions

In this paper, two different slack bus models for unbalanced distribution power flow are reviewed: a single slack bus model and a network-based distributed slack model. Slack bus models which can quantify loads and losses are directly linked to cost analysis approaches for DG installations. Detailed mathematical expressions for cost analysis were presented and discussed. The cost analysis method based on the distributed slack bus model can be used to balance the benefits between utilities and DG operators through setting proper loss contribution rates.

Following MATLAB implementations of the model and cost analysis procedures, simulation results on a system derived from an actual distribution network show that different slack bus models may cause significantly different results in operating costs. In addition, studies with respect to DG penetration levels, DG locations, and loss payment rates show that distributing losses and encouraging DG operators to participate can positively benefit both utilities and DG owners. As a consequence, integrating a distributed slack bus approach can assist both utilities and DG owners to improve their cost evaluations and to better determine the potential benefits of planned installations and those under consideration. Thus, the methodologies can be integrated into rigorous optimization schemes for optimal DG placement.

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